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REPRESENTATION OF SOLUTIONS OF A SECOND-ORDER SYSTEM OF TWO DIFFERENCE EQUATIONS WITH VARIABLE COEFFICIENTS

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ABSTRACT. A definition of system of two nonlinear difference equations with variable coefficients is given. Our main result shows that the difference equation is solvable in closed form and thus for the constant coefficients. Some applications of the main result are also given.

1. INTRODUCTION

The nonlinear difference equations and systems have been considered in a number of papers recently (cf. in particular [1]- [14] and the references cited therein). However, there are some classical classes of solvable difference equations and methods for solving them can be found (see, for example, [8], [9], [12], [16], [17], [18], [19]). In particular, Stević [15] gave some additional information on the behavior of the solutions of the following difference equation

$$x_{n+1} = \frac{x_{n-1}}{1 + x_n x_{n-1}}, n \in \mathbb{N}_0.$$

In Clark and Kulenovic [7] investigated the global asymptotic stability and asymptotic behavior of the following system

$$x_{n+1} = \frac{x_n}{a + cy_n}, y_{n+1} = \frac{y_n}{b + dx_n}, n \in \mathbb{N}_0.$$

Elsayed [8] has got the solutions of the following systems of the difference equations

$$x_{n+1} = \frac{x_{n-1}}{\pm 1 + y_n x_{n-1}}, y_{n+1} = \frac{y_{n-1}}{\mp 1 + x_n y_{n-1}}, n \in \mathbb{N}_0.$$

Motivated by all above mentioned work, and especially by [8], here we investigate the form of the solutions of the system of two-dimensional nonlinear difference equations

$$x_{n+1} = \frac{c_n x_{n-1}}{a_n + b_n y_n x_{n-1}}, y_{n+1} = \frac{c_n y_{n-1}}{a_n + b_n x_n y_{n-1}}, n \in \mathbb{N}_0. \tag{1.1}$$

Now, we consider system (1.1) in the case when a $c_n \neq 0$ for all $n \in \mathbb{N}_0$. Noticing that in this case, system (1.1) can be written in the form

$$x_{n+1} = \frac{x_{n-1}}{\widehat{a}_n + \widehat{b}_n y_n x_{n-1}}, y_{n+1} = \frac{y_{n-1}}{\widehat{a}_n + \widehat{b}_n x_n y_{n-1}}, n \in \mathbb{N}_0.$$

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where $\widehat{a}_n = \dfrac{a_n}{c_n}$ and $\widehat{b}_n = \dfrac{b_n}{c_n}$, for all $n \in \mathbb{N}_0$, we see that we may assume that $c_n = 1$, for all $n \in \mathbb{N}_0$. Hence we consider, without loss of generality, the system

$$x_{n+1} = \frac{x_{n-1}}{a_n + b_n y_n x_{n-1}}, y_{n+1} = \frac{y_{n-1}}{a_n + b_n x_n y_{n-1}}, n \in \mathbb{N}_0. \tag{1.2}$$

using the same notation for coefficients as in (1.1) except for the coefficients c_n, assuming that $c_n = 1$, for all $n \in \mathbb{N}_0$.

2. Main results

Assume that $\{x_n, y_n\}$ is a well-defined solution to system (1.2). In this section, we investigate the solutions of the two-dimensional system of rational difference equations (1.2). Following the idea in Baštinec et al. [3], we use a transformation which reduces rational system (1.2) to system of nonhomogeneous linear difference equations. If we multiply the first equation in system (1.2) by y_n, the second by x_n, and then use such obtained system to change the variables

$$u_n = \frac{1}{y_n x_{n-1}}, v_n = \frac{1}{x_n y_{n-1}},\ n \in \mathbb{N}_0. \tag{2.1}$$

the system is, for $n \in \mathbb{N}_0$, transform into,

$$\begin{cases} u_{n+1} = a_n v_n + b_n \\ v_{n+1} = a_n u_n + b_n \end{cases}. \tag{2.2}$$

System (2.2) implies that for $n \geq 1$,

$$\begin{cases} u_{n+1} = a_n a_{n-1} u_{n-1} + a_n b_{n-1} + b_n \\ v_{n+1} = a_n a_{n-1} v_{n-1} + a_n b_{n-1} + b_n \end{cases}, \tag{2.3}$$

where values for u_0, v_0 are computed by (2.1) with $n = 0$. System (2.3) implies that the sequences $(u_{2n+m})_{n\in\mathbb{N}_0}$ and $(v_{2n+m})_{n\in\mathbb{N}_0}$, $m \in \{0,1\}$, are solutions of the system of linear difference equation

$$\begin{cases} u_{2n} = \left\{\prod_{i=1}^{2} a_{2n-i}\right\} u_{2(n-1)} + \sum_{r=1}^{2} \left\{\prod_{i=1}^{r-1} a_{2n-i}\right\} b_{2n-r} \\ u_{2n+1} = \left\{\prod_{i=1}^{2} a_{2n+1-i}\right\} u_{2(n-1)+1} + \sum_{r=1}^{2} \left\{\prod_{i=1}^{r-1} a_{2n+1-i}\right\} b_{2n+1-r} \\ v_{2n} = \left\{\prod_{i=1}^{2} a_{2n-i}\right\} v_{2(n-1)} + \sum_{r=1}^{2} \left\{\prod_{i=1}^{r-1} a_{2n-i}\right\} b_{2n-r} \\ v_{2n+1} = \left\{\prod_{i=1}^{2} a_{2n+1-i}\right\} v_{2(n-1)+1} + \sum_{r=1}^{2} \left\{\prod_{i=1}^{r-1} a_{2n+1-i}\right\} b_{2n+1-r} \end{cases},$$

where $\prod_{i=1}^{r} a_i = 1$ if $r < 1$. Thus, we have that the general solution of system (2.2) is

$$\begin{cases} u_{2n} = u_0 \left\{\prod_{s=0}^{n-1}\prod_{i=1}^{2} a_{2(n-s)-i}\right\} + \sum_{r=0}^{n-1} \left\{\prod_{s=0}^{r-1}\prod_{i=1}^{2} a_{2(n-s)-i}\right\} \left(\sum_{t=1}^{2} \left\{\prod_{i=1}^{t-1} a_{2(n-r)-i}\right\} b_{2(n-r)-t}\right) \\ u_{2n+1} = u_1 \left\{\prod_{s=0}^{n-1}\prod_{i=1}^{2} a_{2(n-s)+1-i}\right\} + \sum_{r=0}^{n-1} \left\{\prod_{s=0}^{r-1}\prod_{i=1}^{2} a_{2(n-s)+1-i}\right\} \left(\sum_{t=1}^{2} \left\{\prod_{i=1}^{t-1} a_{2(n-r)+1-i}\right\} b_{2(n-r)+1-t}\right) \\ v_{2n} = v_0 \left\{\prod_{s=0}^{n-1}\prod_{i=1}^{2} a_{2(n-s)-i}\right\} + \sum_{r=0}^{n-1} \left\{\prod_{s=0}^{r-1}\prod_{i=1}^{2} a_{2(n-s)-i}\right\} \left(\sum_{t=1}^{2} \left\{\prod_{i=1}^{t-1} a_{2(n-r)-i}\right\} b_{2(n-r)-t}\right) \\ v_{2n+1} = v_1 \left\{\prod_{s=0}^{n-1}\prod_{i=1}^{2} a_{2(n-s)+1-i}\right\} + \sum_{r=0}^{n-1} \left\{\prod_{s=0}^{r-1}\prod_{i=1}^{2} a_{2(n-s)+1-i}\right\} \left(\sum_{t=1}^{2} \left\{\prod_{i=1}^{t-1} a_{2(n-r)+1-i}\right\} b_{2(n-r)+1-t}\right) \end{cases},$$

for all $n \in \mathbb{N}$ and $m \in \{0,1\}$, where $\sum_{i=1}^{r} a_i = 0$ if $r < 1$. The following theorem gives us the main result for system of difference equations (2.2).

Theorem 2.1. *Let $\{u_n, v_n\}_{n\geq 0}$ be solutions of system (2.2). Then $\{u_n\}_{n\geq 0}$ and $\{v_n\}_{n\geq 0}$ are given by the formulas for $n = 0, 1, ...$*

(2.4)

$$\{u_n\}_{n\geq 0} : \begin{cases} u_{2n} = u_0 \left\{ \prod_{s=0}^{n-1} \prod_{i=1}^{2} a_{2(n-s)-i} \right\} + \sum_{r=0}^{n-1} \left\{ \prod_{s=0}^{r-1} \prod_{i=1}^{2} a_{2(n-s)-i} \right\} \left(\sum_{t=1}^{2} \left\{ \prod_{i=1}^{t-1} a_{2(n-r)-i} \right\} b_{2(n-r)-t} \right) \\ u_{2n+1} = (a_0 v_0 + b_0) \\ \times \left\{ \prod_{s=0}^{n-1} \prod_{i=1}^{2} a_{2(n-s)+1-i} \right\} + \sum_{r=0}^{n-1} \left\{ \prod_{s=0}^{r-1} \prod_{i=1}^{2} a_{2(n-s)+1-i} \right\} \left(\sum_{t=1}^{2} \left\{ \prod_{i=1}^{t-1} a_{2(n-r)+1-i} \right\} b_{2(n-r)+1-t} \right) \end{cases},$$

and

(2.5)

$$\{v_n\}_{n\geq 0} : \begin{cases} v_{2n} = v_0 \left\{ \prod_{s=0}^{n-1} \prod_{i=1}^{2} a_{2(n-s)-i} \right\} + \sum_{r=0}^{n-1} \left\{ \prod_{s=0}^{r-1} \prod_{i=1}^{2} a_{2(n-s)-i} \right\} \left(\sum_{t=1}^{2} \left\{ \prod_{i=1}^{t-1} a_{2(n-r)-i} \right\} b_{2(n-r)-t} \right) \\ v_{2n+1} = (a_0 u_0 + b_0) \\ \times \left\{ \prod_{s=0}^{n-1} \prod_{i=1}^{2} a_{2(n-s)+1-i} \right\} + \sum_{r=0}^{n-1} \left\{ \prod_{s=0}^{r-1} \prod_{i=1}^{2} a_{2(n-s)+1-i} \right\} \left(\sum_{t=1}^{2} \left\{ \prod_{i=1}^{t-1} a_{2(n-r)+1-i} \right\} b_{2(n-r)+1-t} \right) \end{cases}.$$

Corollary 2.2. In the constant case, i.e., when the coefficients are constants ($a_n = a$ and $b_n = b$ for all $n \in \mathbb{N}$), in the Theorem (2.1), the solution $(2.4) - (2.5)$ reduces to

$$\{u_n\}_{n\geq 0} : \begin{cases} u_{2n} = u_0 a^{2n} + b(1+a) \sum_{r=0}^{n-1} a^{2r} \\ u_{2n+1} = (a_0 v_0 + b_0) a^{2n} + b(1+a) \sum_{r=0}^{n-1} a^{2r} \end{cases},$$

and

$$\{v_n\}_{n\geq 0} : \begin{cases} v_{2n} = v_0 a^{2n} + b(1+a) \sum_{r=0}^{n-1} a^{2r} \\ v_{2n+1} = (a_0 u_0 + b_0) a^{2n} + b(1+a) \sum_{r=0}^{n-1} a^{2r} \end{cases}.$$

Now note that from (2.1) we have

$$x_n = \frac{u_{n-1}}{v_n} x_{n-2} \text{ and } y_n = \frac{v_{n-1}}{u_n} y_{n-2},$$

from which it follows that

$$\begin{cases} x_{2n} = \frac{u_{2n-1}}{v_{2n}} x_{2(n-1)} \\ x_{2n+1} = \frac{u_{2n}}{v_{2n+1}} x_{2(n-1)+1} \end{cases} \text{ and } \begin{cases} y_{2n} = \frac{v_{2n-1}}{u_{2n}} y_{2(n-1)} \\ y_{2n+1} = \frac{v_{2n}}{u_{2n+1}} y_{2(n-1)+1} \end{cases}, \text{ for all } n \in \mathbb{N}_0.$$

On the other hand, we get explicit solutions of system (1.2), for all $n \in \mathbb{N}_0$,

$$\begin{cases} x_{2n} = x_0 \left\{ \prod_{k=0}^{n-1} \frac{u_{2(n-k)-1}}{v_{2(n-k)}} \right\} \\ x_{2n+1} = x_1 \left\{ \prod_{k=0}^{n-1} \frac{u_{2(n-k)}}{v_{2(n-k)+1}} \right\} \end{cases} \text{ and } \begin{cases} y_{2n} = y_0 \left\{ \prod_{k=0}^{n-1} \frac{v_{2(n-k)-1}}{u_{2(n-k)}} \right\} \\ y_{2n+1} = y_1 \left\{ \prod_{k=0}^{n-1} \frac{v_{2(n-k)}}{u_{2(n-k)+1}} \right\} \end{cases}.$$

Hence we have the following result.

Theorem 2.3. *Let $\{x_n, y_n\}_{n\geq -1}$ be solutions of system (1.2). Then $\{x_n\}_{n\geq -1}$ and $\{y_n\}_{n\geq -1}$ are given by the formulas for $n = 0, 1, ...$*

$$\begin{cases} x_{2n} = x_0 (a_0 + b_0 x_0 y_{-1})^n \prod_{k=0}^{n-1} A_{n,k} \\ x_{2n+1} = x_{-1} (a_0 + b_0 y_0 x_{-1})^{-n-1} \prod_{k=0}^{n-1} B_{n,k} \end{cases}, \quad (2.6)$$

and

$$
\text{(2.7)} \qquad \begin{cases} y_{2n} = y_0\left(a_0 + b_0 y_0 x_{-1}\right)^n \prod\limits_{k=0}^{n-1} A_{n,k} \\ y_{2n+1} = y_{-1}\left(a_0 + b_0 x_0 y_{-1}\right)^{-n-1} \prod\limits_{k=0}^{n-1} B_{n,k} \end{cases}
$$

where

$$
\begin{aligned}
A_{n,k} &= \frac{\left\{\prod\limits_{s=0}^{n-k-2}\prod\limits_{i=1}^{2} a_{2(n-k-s)-1-i}\right\} + C_{n,k}}{\left\{\prod\limits_{s=0}^{n-k-1}\prod\limits_{i=1}^{2} a_{2(n-k-s)-i}\right\} + D_{n,k}}, \\
B_{n,k} &= \frac{\left\{\prod\limits_{s=0}^{n-k-1}\prod\limits_{i=1}^{2} a_{2(n-k-s)-i}\right\} + D_{n,k}}{\left\{\prod\limits_{s=0}^{n-k-1}\prod\limits_{i=1}^{2} a_{2(n-k-s)+1-i}\right\} + E_{n,k}}, \\
C_{n,k} &= \sum_{r=0}^{n-k-2}\left\{\prod_{s=0}^{r-1}\prod_{i=1}^{2} a_{2(n-k-s)-1-i}\right\}\left(\sum_{t=1}^{2}\left\{\prod_{i=1}^{t-1} a_{2(n-k-r)-1-i}\right\} b_{2(n-k-r)-1-t}\right), \\
D_{n,k} &= \sum_{r=0}^{n-k-1}\left\{\prod_{s=0}^{r-1}\prod_{i=1}^{2} a_{2(n-k-s)-i}\right\}\left(\sum_{t=1}^{2}\left\{\prod_{i=1}^{t-1} a_{2(n-k-r)-i}\right\} b_{2(n-k-r)-t}\right), \\
E_{n,k} &= \sum_{r=0}^{n-k-1}\left\{\prod_{s=0}^{r-1}\prod_{i=1}^{2} a_{2(n-k-s)+1-i}\right\}\left(\sum_{t=1}^{2}\left\{\prod_{i=1}^{t-1} a_{2(n-k-r)+1-i}\right\} b_{2(n-k-r)+1-t}\right).
\end{aligned}
$$

Corollary 2.4. In the constant case, in the Theorem (2.3), the solution $(2.6)-(2.7)$ reduces to

$$
\{x_n\}_{n\geq 0} : \begin{cases} x_{2n} = x_0\left(a_0 + b_0 x_0 y_{-1}\right)^n \left(a^{2(n+1)} + b\left(1+a\right)\sum\limits_{r=1}^{n} a^{2r}\right)^{-1} \\ x_{2n+1} = x_{-1}\left(a + b y_0 x_{-1}\right)^{-n-1} \end{cases}
$$

and

$$
\{y_n\}_{n\geq 0} : \begin{cases} y_{2n} = y_0\left(a_0 + b_0 y_0 x_{-1}\right)^n \left(a^{2(n+1)} + b\left(1+a\right)\sum\limits_{r=1}^{n} a^{2r}\right)^{-1} \\ y_{2n+1} = y_{-1}\left(a + b x_0 y_{-1}\right)^{-n-1} \end{cases}.
$$

Remark 2.5. In this remark we use the formulae in Theorem 2.3 to get solutions of system (1.1), when $c_n \neq 0$ for $n \in \mathbb{N}_0$. So, we replace sequences $(a_n)_{n\in\mathbb{N}_0}$ and $(b_n)_{n\in\mathbb{N}_0}$ in formulas of Theorem 2.3 with sequences $\left(\frac{a_n}{c_n}\right)_{n\in\mathbb{N}_0}$ and $\left(\frac{b_n}{c_n}\right)_{n\in\mathbb{N}_0}$.

Remark 2.6. The solutions of the one-dimensional nonlinear rational difference equation

$$
x_{n+1} = \frac{c_n x_{n-1}}{a_n + b_n x_n x_{n-1}}, n \in \mathbb{N}_0,
$$

can be obtained from system (1.1) by taking $x_{-i} = y_{-i}$, $i \in \{0,1\}$.

Example 2.7. We consider interesting numerical example for the difference equations system (1.1) with the initial conditions $x_{-1} = -2$, $x_0 = -0.6$, $y_{-1} = 0.6$ and $y_0 = 2$. Moreover, choosing the sequences $a_n = e^{n+1}$, $b_n = \ln(n+2)$ and $c_n = n+3$, the system (1.1) can be written as follows:

$$
\text{(2.8)} \qquad x_{n+1} = \frac{(n+3)\,x_{n-1}}{e^{n+1} + \ln(n+2)\,y_n x_{n-1}}, y_{n+1} = \frac{(n+3)\,y_{n-1}}{e^{n+1} + \ln(n+2)\,x_n y_{n-1}},
$$

$n = 0, 1, ...$ The plot of the system (2.8) is shown in Figure 1.

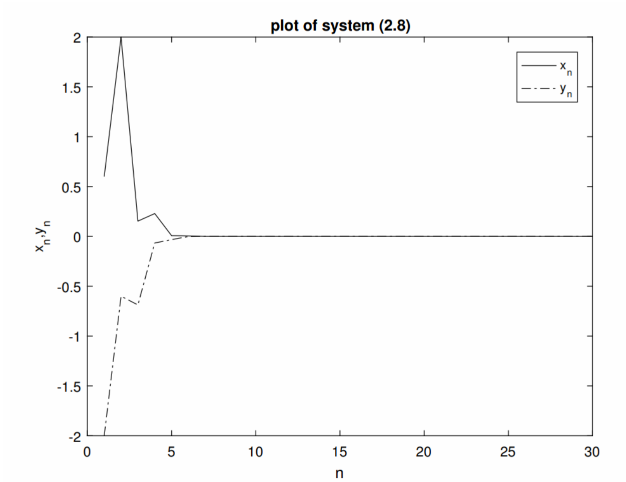

Figure 1.This figure shows the solutions of the system (2.8), when we put

the initial conditions $x_{-1} = -2, x_0 = -0.6, y_{-1} = 0.6$ and $y_0 = 2$.

Example 2.8. We consider interesting numerical example for the difference equations system (1.1) with the initial conditions $x_{-1} = -2$, $x_0 = 0.2$, $y_{-1} = -0.2$ and $y_0 = -5.4$. Moreover, choosing the sequences $a_n = 0.2$, $b_n = 0.45$ and $c_n = 1$, the system (1.1) can be written as follows:

$$x_{n+1} = \frac{x_{n-1}}{0.2 + 0.45 y_n x_{n-1}}, y_{n+1} = \frac{y_{n-1}}{0.2 + 0.45 x_n y_{n-1}}, \tag{2.9}$$

$n = 0, 1, ...$ The plot of the system (2.9) is shown in Figure 2.

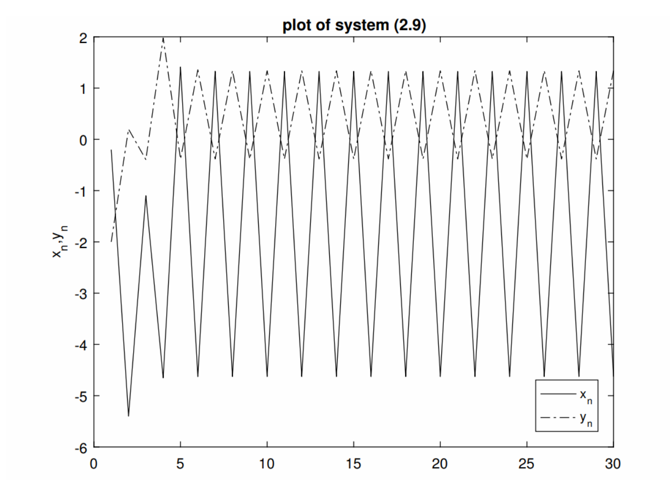

Figure 2. This figure shows the solutions of the system (2.9), when we put

the initial conditions $x_{-1} = -2, x_0 = 0.2, y_{-1} = -0.2$ and $y_0 = -5.4$.

Corollary 2.9. In this corollary, we summarize the solution of system (1.1) in some particular cases

The case			Formulas for well-defined solutions of system (1.1)
$c_n = 0$ for all $n \in \mathbb{N}_0$			$x_n = y_n = 0, n \in \mathbb{N}$
$c_n \neq 0$ for all $n \in \mathbb{N}_0$	$x_{n_0} = 0$ for some $n_0 \in \mathbb{N}$	$x_{-1} = 0$	$x_{2n+1} = 0, y_{2n} = y_0 \left\{ \prod_{i=1}^{n} \frac{c_{2(n-i)+1}}{a_{2(n-i)+1}} \right\}, n \in \mathbb{N}$
		$x_0 = 0$	$x_{2n} = 0, y_{2n+1} = y_{-1} \left\{ \prod_{i=0}^{n} \frac{c_{2(n-i)}}{a_{2(n-i)}} \right\}, n \in \mathbb{N}$
	$y_{n_1} = 0$ for some $n_1 \in \mathbb{N}$	$y_{-1} = 0$	$x_{2n} = x_0 \left\{ \prod_{i=1}^{n} \frac{c_{2(n-i)+1}}{a_{2(n-i)+1}} \right\}, y_{2n+1} = 0, n \in \mathbb{N}$
		$y_0 = 0$	$x_{2n+1} = x_{-1} \left\{ \prod_{i=0}^{n} \frac{c_{2(n-i)}}{a_{2(n-i)}} \right\}, y_{2n} = 0, n \in \mathbb{N}$

Table 1 : Formulas for well-defined solutions of system (1.1) for certain cases.

3. Conclusion

In this paper, we have consider the following two nonlinear difference equations with variable coefficients,

$$x_{n+1} = \frac{c_n x_{n-1}}{a_n + b_n y_n x_{n-1}}, y_{n+1} = \frac{c_n y_{n-1}}{a_n + b_n x_n y_{n-1}}, n \in \mathbb{N}_0,$$

where the sequences (a_n), (b_n), (c_n) and initial values x_{-i}, y_{-i}, $i \in \{0, 1\}$ are non-zero real numbers, for all $n \in \mathbb{N}_0$. We have obtained the explicit form of solutions of the aforementioned system using homogeneous linear difference equation to variable coefficients associated to the system. In particular, we have also obtained the closed-form of well-defined solutions of the two-dimensional systems of nonlinear rational difference equations with constant coefficients. The aforementioned two-dimensional system can extend to the three (resp. higher)-dimensional system of difference equations which is variable coefficients or constant coefficients as special cases.

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