

STABILITY ANALYSIS OF A BANK DEPOSIT-LOAN MODEL WITH RESERVE REQUIREMENT POLICY

ONIK FEBRIA DAMAYANTI, ISNANI DARTI*, AGUS SURYANTO, AND NUR SHOFIANAH

ABSTRACT. This study investigates the stability of a dynamic loan–deposit model within a single-bank system, incorporating the effects of reserve requirement policies. Unlike existing deposit–loan models, the proposed framework explicitly distinguishes between productive and consumptive loans, allowing a more detailed analysis of credit allocation under regulatory constraints. The model is represented by a system of nonlinear differential equations describing the interactions between deposit funds and the two types of loans. The analysis focuses on identifying equilibrium points and examining their local stability through Jacobian-based linearization. The results reveal five equilibrium points: the equilibrium without deposit and loan activities, the loan-free equilibrium, the productive loan-free equilibrium, the consumptive loan-free equilibrium, and the full-activity equilibrium. Under the stated parameter conditions, all equilibria are locally stable except the trivial equilibrium, which is unstable. Numerical simulations validate the analytical results and illustrate the influence of reserve requirements on deposit dynamics and loan composition. These findings suggest that reserve requirement policies play a crucial role as a regulatory tool in maintaining banking liquidity and promoting a stable balance between productive and consumptive lending.

1. INTRODUCTION

Banking systems are fundamental to financial stability, primarily mobilizing deposits and allocating credit for both productive and consumptive purposes [1]. However, this intermediation involves inherent risks; instability in a single institution can trigger contagion and potentially lead to a systemic crisis [11]. To address these risks, central banks implement reserve requirement (RR) policies, referred to in Indonesia as Giro Wajib Minimum (GWM), as a key regulatory tool [7]. By requiring banks to hold a specified fraction of customer deposits as reserves [9, 15], these policies safeguard liquidity [3], regulate the money supply, and enhance financial resilience [2].

In theory, the reserve requirement (RR) functions as a control on credit expansion: increasing the ratio restricts lending capacity [8], while decreasing it can stimulate credit growth [12]. In Indonesia, where the GWM is set at approximately 9% of third-party funds (DPK) for 2025 [5], the policy directly affects lending capacity by diverting mobilized funds into reserves. Consequently, the RR fulfills a dual role as both a monetary stabilization instrument and a mechanism for mitigating liquidity risk.

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Mathematical modeling provides a rigorous approach for analyzing these complex dynamics. The literature has progressed from classic deposit-loan models to frameworks that incorporate depositor behavior and interbank interactions. The relationship between deposit and loan volumes has been characterized using the Lotka-Volterra predator-prey framework [14]. This approach was subsequently expanded to a single-bank, two-loan system using an analogy with a two-predator one-prey model [6, 10]. In this specific context, loans are represented as predators that interact with deposits, which serve as the prey. However, most existing models treat loans as a homogeneous variable. They seldom differentiate between productive and consumptive loans, and rarely analyze how RR policies influence the stability of these distinct credit structures. As a result, the interaction among reserve requirements, loan composition, and banking stability remains insufficiently explored.

This study addresses this gap by developing a nonlinear differential equation model for a single-bank system that explicitly distinguishes between productive and consumptive loans. This dual-loan structure enables a more detailed analysis of how reserve requirements affect liquidity management and portfolio composition. The research identifies equilibrium points, examines their local stability using the Jacobian matrix, and validates the analytical results through numerical simulations to assess how RR policies can maintain a balance between operational efficiency and systemic stability.

The structure of the paper is as follows: Section 2 presents the model formulation; Section 3 provides the dynamical analysis, including the identification of equilibrium points and the examination of their local stability; Section 4 reports the numerical simulations; and Section 5 concludes the study with final remarks.

2. MODEL FORMULATION

This study formulates a nonlinear differential equation model, as presented in (2.1), to capture the dynamic interactions between deposit funds (D) and loan allocations under the influence of reserve requirement policies. This model is developed from the model in [6], which captures the essential mechanisms of deposit accumulation and loan distribution. It distinguishes between consumptive loans (L_1) and productive loans (L_2) to provide a more realistic representation of the banking system while incorporating the reserve requirement parameter (m).

$$(2.1) \quad \begin{aligned} \frac{dD}{dt} &= \gamma D \left(1 - \frac{D}{k}\right) - \frac{p_1(1-m)DL_1}{b_1(1-m)D+1} - \frac{p_2(1-m)DL_2}{b_2(1-m)D+1} - wD, \\ \frac{dL_1}{dt} &= \frac{p_1(1-m)DL_1}{b_1(1-m)D+1} - H_5 L_1 - \beta_1 L_1 L_2 - h_1 L_1^2, \\ \frac{dL_2}{dt} &= \frac{p_2(1-m)DL_2}{b_2(1-m)D+1} - H_6 L_2 - \beta_2 L_1 L_2 - h_2 L_2^2, \end{aligned}$$

where

$$\begin{aligned} H_5 &= \sigma_1 + n_1 + c_1(1 - \mu_1), \\ H_6 &= \sigma_2 + n_2 + c_2(1 - \mu_2), \end{aligned}$$

with nonnegative initial conditions $D(0)$, $L_1(0)$, $L_2(0)$, and with $\gamma > w$. The parameter descriptions for model (2.1) are presented in Table 1, where $i = 1$ denotes the consumptive loans and $i = 2$ denotes the productive loans. This model extends the model in [6] through the inclusion of the reserve requirement parameter, and the analysis partly follows that framework.

TABLE 1. Description of Parameters

Parameter	Description
b_i	Borrowing constraint
c_i	Loan repayment rate
h_1	Competition rate among consumptive loans
h_2	Competition rate among productive loans
k	Bank's deposit capacity
m	Reserve requirement ratio
n_i	Non-performing loan rate
p_i	Maximum proportion of deposits allocated to loans
w	Deposit withdrawal rate
β_1	Competition rate affecting consumptive loans
β_2	Competition rate affecting productive loans
γ	Deposit inflow rate driven by interest rate
μ_i	Proportion of non-performing loans (NPLs)
σ_i	Loan interest rate

The functional forms in system (2.1) are economically motivated by standard assumptions in dynamic banking models. The deposit equation follows a logistic growth structure to reflect natural deposit accumulation with capacity constraints, while withdrawals are represented by a linear decay term. The interaction terms between deposits and loans adopt a saturating response function, capturing the limited ability of banks to transform deposits into loans due to operational frictions and regulatory constraints. The reserve requirement parameter m reduces the effective deposits available for lending, reflecting the role of central bank regulation in restricting loan expansion and mitigating liquidity risk [14, 15].

3. DYNAMICAL ANALYSIS

3.1. The equilibrium points. The equilibrium points are determined by setting all time derivatives in system (1) equal to zero. This procedure yields five possible equilibrium states, which are described below.

- (1) The trivial equilibrium, $E_0 = (0, 0, 0)$, representing the absence of both deposit and loan activities, which exists unconditionally.
- (2) The loan-free equilibrium point, $E_1 = (D_{(1)}, 0, 0)$ where $D_{(1)} = \frac{k}{\gamma}(\gamma - w)$. This equilibrium always exists since the condition $\gamma > w$ holds.

- (3) The consumptive loan-free equilibrium point, $E_2 = (D_{(2)}, 0, L_{2(2)})$, which exists if $H_{4(2)} > H_6$, where

$$L_{2(2)} = \frac{1}{h_2} \left(\frac{p_2(1-m)D_{(2)}}{1+b_2(1-m)D_{(2)}} - H_6 \right),$$

$$H_{4(2)} = \frac{p_2(1-m)D_{(2)}}{1+b_2(1-m)D_{(2)}}.$$

The equilibrium point E_2 exists provided that the corresponding cubic equation for $D_{(2)}$ admits a real and positive solution. This condition is verified using the Cardan criterion [4], following the approach in [6].

- (4) The productive loan-free equilibrium point, $E_3 = (D_{(3)}, L_{1(3)}, 0)$, which exists if $H_{3(3)} > H_5$, where

$$L_{1(3)} = \frac{1}{h_1} \left(\frac{p_1(1-m)D_{(3)}}{1+b_1(1-m)D_{(3)}} - H_5 \right),$$

$$H_{3(3)} = \frac{p_1(1-m)D_{(3)}}{1+b_1(1-m)D_{(3)}}.$$

The equilibrium point E_3 exists provided that the corresponding cubic equation for $D_{(3)}$ admits a real and positive solution. This condition is verified using the Cardan criterion, as demonstrated in [6].

- (5) The equilibrium point corresponding to the coexistence of deposits and loans is denoted by $E^* = (D^*, L_1^*, L_2^*)$, where

$$L_1^* = \frac{s_1 + s_3[\beta_1 H_6 - H_5 h_2]}{s_3(h_1 h_2 - \beta_1 \beta_2)}, L_2^* = \frac{s_2 + s_3[\beta_2 H_5 - H_6 h_1]}{s_3(h_1 h_2 - \beta_1 \beta_2)}$$

$$\begin{aligned} s_1 &= p_1(1-m)h_2(D^* + b_2(1-m)D^{*2}) - \beta_1 p_2(1-m)(D^* + b_1(1-m)D^{*2}), \\ s_2 &= p_2(1-m)h_1(D^* + b_1(1-m)D^{*2}) - \beta_2 p_1(1-m)(D^* + b_2(1-m)D^{*2}), \\ s_3 &= (1 + b_1(1-m)D^*)(1 + b_2(1-m)D^*). \end{aligned}$$

The existence of L_1^* and L_2^* is guaranteed if at least one of these conditions holds

- (a) If $h_1 h_2 > \beta_1 \beta_2$, then it must satisfy

$$\frac{h_2}{\beta_1} > \frac{H_4^* - H_6}{H_3^* - H_5} > \frac{\beta_2}{h_1}.$$

- (b) If $h_1 h_2 < \beta_1 \beta_2$, then it must satisfy

$$\frac{h_2}{\beta_1} < \frac{H_4^* - H_6}{H_3^* - H_5} < \frac{\beta_2}{h_1}.$$

Furthermore, the existence of D^* will be established through the numerical simulation.

3.2. Local stability analysis. To determine the stability properties of every equilibrium point, we utilize local stability analysis through linearization of the system (2.1) using the Jacobian matrix as follows.

$$(3.1) \quad J(D, L_1, L_2) = \begin{pmatrix} J_1 & -H_3 & -H_4 \\ H_1 & J_2 & -\beta_1 L_1 \\ H_2 & -\beta_2 L_2 & J_3 \end{pmatrix},$$

where

$$\begin{aligned} J_1 &= \gamma \left(1 - \frac{2D}{k} \right) - H_1 - H_2 - w, \\ J_2 &= H_3 - H_5 - \beta_1 L_2 - 2h_1 L_1, \\ J_3 &= H_4 - H_6 - \beta_2 L_1 - 2h_2 L_2, \end{aligned}$$

$$\begin{aligned} H_1 &= \frac{p_1(1-m)L_1}{(b_1(1-m)D+1)^2}, H_2 = \frac{p_2(1-m)L_2}{(b_2(1-m)D+1)^2}, \\ H_3 &= \frac{p_1(1-m)D}{b_1(1-m)D+1}, H_4 = \frac{p_2(1-m)D}{b_2(1-m)D+1}. \end{aligned}$$

The eigenvalues of the Jacobian matrix (3.1) determine the stability of the equilibrium points, as presented below.

Theorem 3.1. *The following results hold.*

- (1) *The equilibrium point E_0 is always unstable.*
- (2) *If $\frac{p_1(1-m)D_{(1)}}{b_1(1-m)D_{(1)}+1} < H_5$ and $\frac{p_2(1-m)D_{(1)}}{b_2(1-m)D_{(1)}+1} < H_6$, the loan-free equilibrium point, E_1 , is locally asymptotically stable (LAS). Otherwise, E_1 is unstable.*

Proof. The proof is divided into two parts.

$$(1) \text{ The Jacobian matrix at } E_0 \text{ is } J(0,0,0) = \begin{pmatrix} \gamma - w & 0 & 0 \\ 0 & -H_5 & 0 \\ 0 & 0 & -H_6 \end{pmatrix} \text{ and its eigenvalues are}$$

$$\begin{aligned} \lambda_1 &= \gamma - w, \\ \lambda_2 &= -H_5 = -\sigma_1 - n_1 - c_1(1 - \mu_1), \\ \lambda_3 &= -H_6 = -\sigma_2 - n_2 - c_2(1 - \mu_2). \end{aligned}$$

Since $\gamma > w$, it follows that $\lambda_1 > 0$. Therefore, the equilibrium point E_0 , representing the absence of both deposit and loan activities, is necessarily unstable.

- (2) The Jacobian matrix at E_1 is given by

$$J(E_1) = \begin{pmatrix} -(\gamma - w) & -\frac{p_1(1-m)D_{(1)}}{b_1(1-m)D_{(1)}+1} & -\frac{p_2(1-m)D_{(1)}}{b_2(1-m)D_{(1)}+1} \\ 0 & \frac{p_1(1-m)D_{(1)}}{b_1(1-m)D_{(1)}+1} - H_5 & 0 \\ 0 & 0 & \frac{p_2(1-m)D_{(1)}}{b_2(1-m)D_{(1)}+1} - H_6 \end{pmatrix}.$$

The eigenvalues of the Jacobian matrix at E_1 are

$$\begin{aligned} \lambda_1 &= -(\gamma - w), \\ \lambda_2 &= \frac{p_1(1-m)D_{(1)}}{b_1(1-m)D_{(1)}+1} - H_5, \\ \lambda_3 &= \frac{p_2(1-m)D_{(1)}}{b_2(1-m)D_{(1)}+1} - H_6. \end{aligned}$$

It is clear that $\lambda_1 < 0$. The eigenvalues λ_2 and λ_3 are negative if

$$(3.2) \quad \frac{p_1(1-m)D_{(1)}}{b_1(1-m)D_{(1)}+1} < H_5 \text{ and } \frac{p_2(1-m)D_{(1)}}{b_2(1-m)D_{(1)}+1} < H_6.$$

Thus, the loan-free equilibrium point E_1 is LAS if the conditions in (3.2) are satisfied.

Furthermore, if these conditions (3.2) are not satisfied, then the equilibrium point E_1 is unstable. □

Theorem 3.2. *If $H_{3(2)} - H_5 < \beta_1 L_{2(2)}$ and $\frac{\gamma}{k} > H_{2(2)} b_2(1 - m)$, then the consumptive loan-free equilibrium point E_2 is LAS, where $H_{3(2)}$ and $H_{2(2)}$ are defined in the proof.*

Proof. The Jacobian matrix at E_2 is

$$J(E_2) = \begin{pmatrix} D_{(2)} \left(H_{2(2)} b_2(1-m) - \frac{\gamma}{k} \right) & -H_{3(2)} & -H_{4(2)} \\ 0 & H_{3(2)} - H_5 - \beta_1 L_{2(2)} & 0 \\ H_{2(2)} & -\beta_2 L_{2(2)} & -h_2 L_{2(2)} \end{pmatrix},$$

where

$$H_{2(2)} = \frac{p_2(1-m)L_{2(2)}}{(b_2(1-m)D_{(2)} + 1)^2}, H_{3(2)} = \frac{p_1(1-m)D_{(2)}}{b_1(1-m)D_{(2)} + 1}, \text{ and } H_{4(2)} = \frac{p_2(1-m)D_{(2)}}{b_2(1-m)D_{(2)} + 1}.$$

The Jacobian matrix $J(E_2)$ yields the characteristic equation as follows

$$(H_{3(2)} - H_5 - \beta_1 L_{2(2)} - \lambda) |J_{(2)} - \lambda I| = 0,$$

where

$$J_{(2)} = \begin{pmatrix} D_{(2)} \left(H_{2(2)} b_2(1-m) - \frac{\gamma}{k} \right) & -H_{4(2)} \\ H_{2(2)} & -h_2 L_{2(2)} \end{pmatrix}.$$

It can be observed that the first eigenvalue is $\lambda_1 = H_{3(2)} - H_5 - \beta_1 L_{2(2)}$ which becomes negative whenever $H_{3(2)} - H_5 < \beta_1 L_{2(2)}$. The remaining eigenvalues λ_2 and λ_3 will be negative if $\text{tr}(J_{(2)}) < 0$ and $|J_{(2)}| > 0$. Moreover, if $\frac{\gamma}{k} > H_{2(2)} b_2(1-m)$, then $\text{tr}(J_{(2)}) < 0$ and $|J_{(2)}| > 0$ are satisfied. Hence, if $H_{3(2)} - H_5 < \beta_1 L_{2(2)}$ and $\frac{\gamma}{k} > H_{2(2)} b_2(1-m)$, then E_2 is LAS. $\square$

Theorem 3.3. If $H_{4(3)} - H_6 < \beta_2 L_{1(3)}$ and $\frac{\gamma}{k} > H_{1(3)} b_1(1-m)$, then the productive loan-free equilibrium point E_3 is LAS, where

$$H_{1(3)} = \frac{p_1(1-m)L_{1(3)}}{(b_1(1-m)D_{(3)} + 1)^2} \text{ and } H_{4(3)} = \frac{p_2(1-m)D_{(3)}}{b_2(1-m)D_{(3)} + 1}.$$

Proof. The proof follows the same line of reasoning as that adopted in Theorem 3.2. $\square$

Theorem 3.4. The deposit-loan coexistence equilibrium point, E^* , is LAS if $a_1 > 0$, $a_3 > 0$, and $a_1 a_2 - a_3 > 0$, where a_1, a_2 , and a_3 are specified in the proof.

Proof. The Jacobian matrix at E^* is given by

$$J(E^*) = \begin{pmatrix} D^* \left(H_1^* b_1(1-m) + H_2^* b_2(1-m) - \frac{\gamma}{k} \right) & -H_3^* & -H_4^* \\ H_1^* & -h_1 L_1^* & -\beta_1 L_1^* \\ H_2^* & -\beta_2 L_2^* & -h_2 L_2^* \end{pmatrix},$$

where

$$H_1^* = \frac{p_1(1-m)L_1^*}{(b_1(1-m)D^* + 1)^2}, \quad H_2^* = \frac{p_2(1-m)L_2^*}{(b_2(1-m)D^* + 1)^2}, \\ H_3^* = \frac{p_1(1-m)D^*}{b_1(1-m)D^* + 1}, \quad H_4^* = \frac{p_2(1-m)D^*}{b_2(1-m)D^* + 1}.$$

The characteristic equation of the matrix $J(E^*)$ is derived as follows:

$$(3.3) \quad \lambda^3 + a_1 \lambda^2 + a_2 \lambda + a_3 = 0,$$

where

$$\begin{aligned} a_1 &= j_2 + j_5 - j_1, \\ a_2 &= j_2 j_5 + H_2^* H_4^* + H_1^* H_3^* - j_1 j_2 - j_1 j_5 - j_3 j_4, \\ a_3 &= j_1 j_3 j_4 + j_2 H_2^* H_4^* + j_5 H_1^* H_3^* - j_1 j_2 j_5 - j_3 H_2^* H_3^* - j_4 H_1^* H_4^*. \end{aligned}$$

$$\begin{aligned} j_1 &= D^* \left(H_1^* b_1 (1-m) + H_2^* b_2 (1-m) - \frac{\gamma}{k} \right), \\ j_2 &= h_1 L_1^*, j_3 = \beta_1 L_1^*, j_4 = \beta_2 L_2^*, j_5 = h_2 L_2^*. \end{aligned}$$

By applying the Routh–Hurwitz criterion [13], the eigenvalues of equation (3.3) have negative real parts if and only if $a_1 > 0$, $a_3 > 0$, and $a_1 a_2 - a_3 > 0$. Hence, E^* is LAS under these conditions. $\square$

4. NUMERICAL SIMULATIONS

This section reports the numerical simulation results of the model (2.1) to validate our theoretical findings. We employed the classical fourth-order Runge-Kutta method to solve the system of ordinary differential equations. This method is also a standard tool for the numerical analysis of dynamical systems.

The parameter values of model (2.1) are summarized in Table 2. All the parameter values are chosen hypothetically because there is currently no data available for our proposed model. For all simulations, the initial condition are set to $D(0) = 3$, $L_1(0) = 1$, and $L_2(0) = 2$. In the first simulation, the parameter values listed in the column for simulation 1 in Table 2 are applied. The results indicate that the system converges asymptotically to the loan-free equilibrium point, $E_1 = (1.5, 0, 0)$. In addition, it was found that

$$\frac{p_1(1-m)D_{(1)}}{b_1(1-m)D_{(1)}+1} = 0.23 < 0.44 = H_5 \text{ and } \frac{p_2(1-m)D_{(1)}}{b_2(1-m)D_{(1)}+1} = 0.25 < 0.58 = H_6.$$

This result indicates that the parameter values also meet the requirements for local stability. Figure 1 illustrates this behavior by showing that the solution converges to E_1 .

TABLE 2. Simulation Parameters

Parameter	Simulation 1	Simulation 2	Simulation 3	Simulation 4
γ	0.4	0.4	0.4	0.4
w	0.2	0.2	0.2	0.2
k	3	3	3	3
m	0.075	0.075	0.075	0.075
p_1	0.25	0.35	0.8	0.85
p_2	0.35	0.8	0.35	0.9
n_1	0.17	0.17	0.17	0.17
n_2	0.23	0.23	0.23	0.2
b_1	0.8	0.8	0.8	0.9
b_2	0.9	0.7	0.9	0.95
σ_1	0.1	0.2	0.1	0.2
σ_2	0.15	0.1	0.15	0.15
c_1	0.2	0.2	0.2	0.15
c_2	0.25	0.2	0.25	0.2
β_1	0.5	0.5	0.5	0.1
β_2	0.4	0.4	0.4	0.15
h_1	0.1	0.1	0.1	0.3
h_2	0.1	0.1	0.1	0.25
μ_1	0.15	0.15	0.15	0.15
μ_2	0.2	0.1	0.2	0.2

Numerical simulation for the equilibrium point E_2 is carried out using the parameter values of simulation 2 presented in Table 2. Based on these parameters, we obtain $H_{4(2)} = 0.52 > 0.51 = H_6$, which implies that the existence condition for $L_{2(2)}$ is satisfied. In addition, the Cardan criterion is also fulfilled, which means that $D_{(2)}$ exists. This is consistent with the calculation, yielding the equilibrium point $E_2 = (1.2773, 0, 0.0733)$. Furthermore, both local stability conditions for E_2 are satisfied, as elaborated below.

$$H_{3(2)} - H_5 = -0.3274 < 0.0367 = \beta_1 L_{2(2)} \text{ and } \frac{\gamma}{k} = 0.1333 > 0.0105 = H_{2(2)} b_2(1 - m).$$

Figure 2 illustrates that the numerical solution converges to the equilibrium point E_2 .

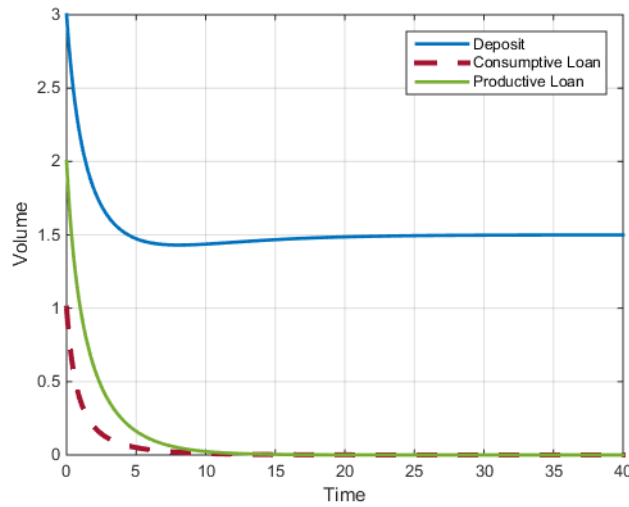


FIGURE 1. Numerical simulation 1.

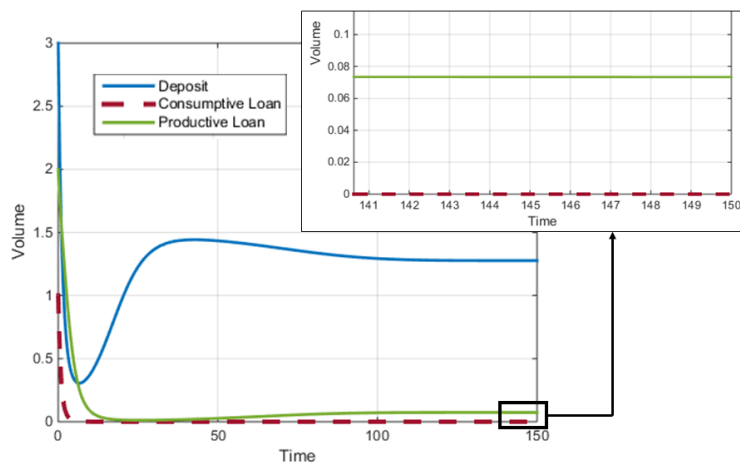


FIGURE 2. Numerical simulation 2.

The numerical simulation for the equilibrium point E_3 is performed using the parameters of Simulation 3 provided in Table 2. From these parameters, it is obtained that

$$H_{3(3)} = 0.45 > 0.44 = H_5,$$

indicating that the existence condition for $L_{1(3)}$ is satisfied. Moreover, one of the Cardan criteria is fulfilled, which ensures the existence of $D_{(3)}$. This result is consistent with the analytical computation of the equilibrium point, namely $E_3 = (1.1173, 0.1250, 0)$. In addition, both local stability conditions for E_3 are verified, as detailed below.

$$H_{4(3)} - H_6 = -0.3926 < 0.0504 = \beta_2 L_{1(3)} \text{ and } \frac{\gamma}{k} = 0.1333 > 0.0201 = H_{1(3)} b_1 (1 - m).$$

Figure 3 illustrates that, starting from the initial conditions, the numerical solution converges to the equilibrium point E_3 .

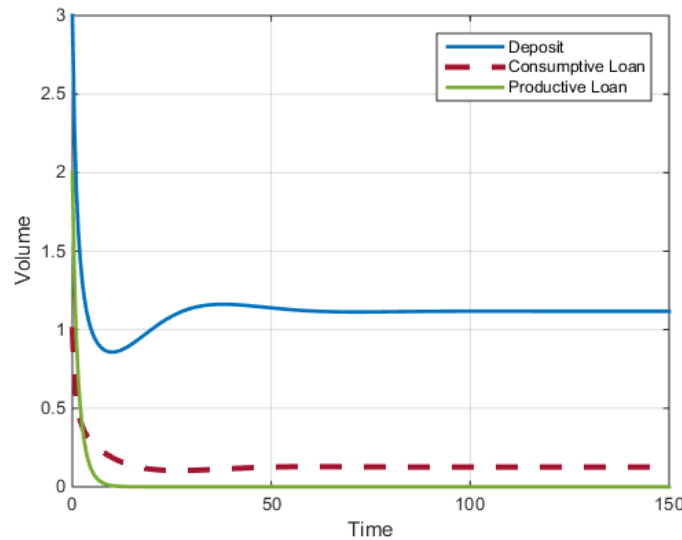


FIGURE 3. Numerical simulation 3.

The parameter values used for the numerical simulation of the equilibrium point E^* are presented in the Simulation 4 column of Table 2. Based on these parameters, we obtain

- (1) $H_4^* = 0.52 > 0.51 = H_6$ and $H_3^* = 0.50 > 0.49 = H_5$,
- (2) $\frac{h_2}{\beta_1} = 2.5 > \frac{H_4^* - H_6}{H_3^* - H_5} = 1.24 > 0.5 = \frac{\beta_2}{h_1}$ in the case $h_1 h_2 = 0.075 > 0.015 = \beta_1 \beta_2$.

This confirms that the existence conditions for both L_1^* and L_2^* are satisfied, leading to the equilibrium point $E^* = (1.3799, 0.0156, 0.0274)$. The local stability of this equilibrium is further verified using the Routh–Hurwitz criterion, as shown below.

- (1) $a_1 = 0.1868 > 0$,
- (2) $a_3 = 0.1565 \times 10^{-4} > 0$, and
- (3) $a_1 a_2 - a_3 = 0.0011 > 0$.

Consequently, in simulation 4, the system trajectories converge toward the equilibrium point E^* , which is illustrated in Figure 4.

The numerical simulations are consistent with the results of the local stability analysis. Whenever the analytical conditions for stability are satisfied, the trajectories of the system converge toward the corresponding equilibrium point. This agreement between analytical derivations and numerical outcomes confirms the validity of the stability criteria, demonstrating that the equilibrium states identified in the model are not only theoretically stable but also observable through dynamic simulations.

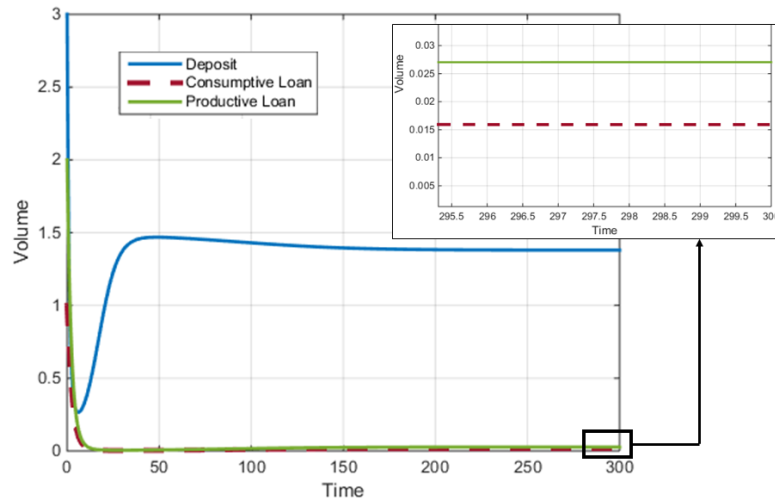


FIGURE 4. Numerical simulation 4.

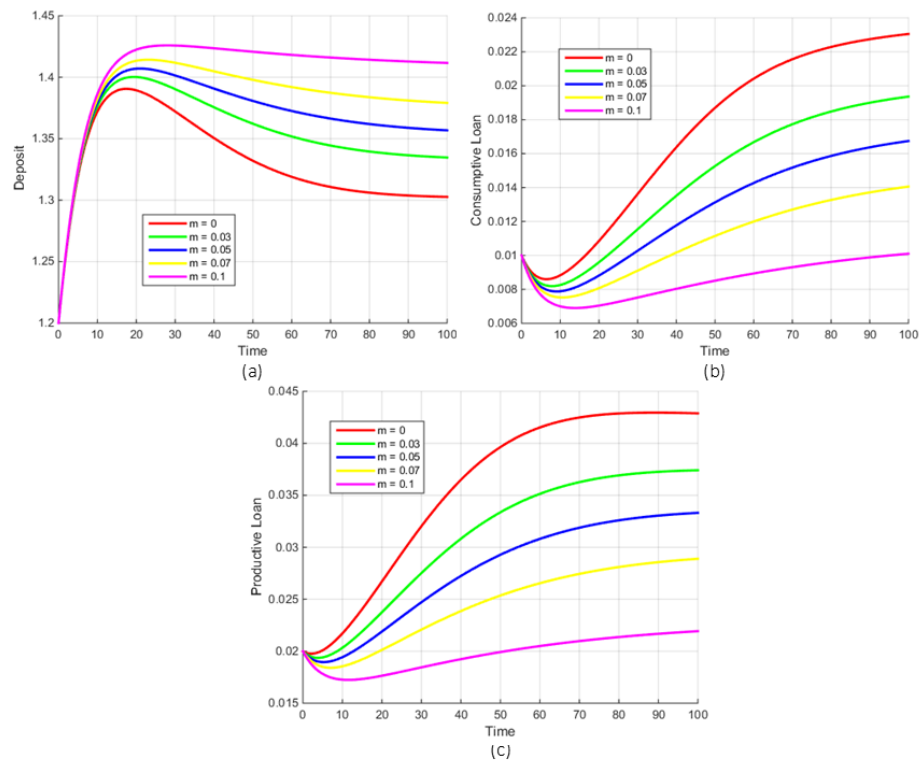


FIGURE 5. Effects of the reserve requirement ratio on (a) deposit volume, (b) consumptive loan volume, and (c) productive loan volume.

Figure 5 illustrates the banking system's responsiveness to variations in the reserve requirement ratio (m). As depicted in Panel (a), an increase in m initially accelerates deposit accumulation. However, the system eventually converges to a lower steady state. This trajectory suggests that while stricter reserve requirements may offer short-term liquidity buffers, they ultimately constrain the long-term accumulation of third-party funds.

A parallel contraction is observed in lending activities. Panel (b) indicates that higher reserve requirements dampen the allocation to consumptive loans. Similarly, Panel (c) reveals a decline in productive loan volumes as m increases, with the impact appearing even more pronounced than in the consumptive sector. Collectively, these simulations underscore the potent role of the reserve requirement as a regulatory throttle moderating available liquidity and reshaping the delicate balance between deposit mobilization and credit allocation.

Our findings highlight the critical function of reserve requirements in stabilizing deposit and loan volumes. The stability analysis reveals that the trivial equilibrium (representing a system collapse) is unstable, which is a mathematically consistent result for a functioning bank. Conversely, the remaining equilibrium points exhibit local asymptotic stability under specific parameter conditions.

Among these, the coexistence equilibrium represents the optimal operational state, reflecting a healthy synergy between deposits, productive loans, and consumptive loans. This equilibrium suggests that a well-calibrated reserve policy can bolster stability without stifling the bank's intermediation function. Here, productive lending secures long-term solvency and repayment capacity, while consumptive lending drives short-term profitability. Together, they form a robust financial structure capable of withstanding regulatory constraints.

The parameter values adopted in the numerical simulations are hypothetical and serve primarily to illustrate the qualitative properties of the model. To strengthen its empirical relevance and predictive reliability, future research should calibrate the model parameters against observed banking data.

5. CONCLUSION

This study developed and analyzed a nonlinear differential equation model to capture the dynamics of a single-bank system, explicitly considering the interactions among deposits, productive loans, and consumptive loans under reserve requirements. Rigorous dynamical analysis identified five equilibrium points. Stability analysis demonstrates that all non-trivial equilibria are locally stable, indicating that the system tends toward a steady state under typical conditions. Numerical simulations support these analytical results, showing that the reserve requirement is a decisive factor in banking dynamics, influencing both deposit levels and loan portfolio composition. The findings suggest that reserve policy is essential for maintaining systemic resilience and directly affects operational efficiency. Future research may extend this framework by exploring global stability properties and incorporating stochastic elements or empirical data to further validate these theoretical insights.

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