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MATRIX THEORY FOR COMPLEX Q-FUZZY SETS AND THEIR APPLICATIONS IN ADMINISTRATIVE DECISION-MAKING PROBLEMS

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ABSTRACT. The complex Q-fuzzy set (CQFS) is a novel method for solving the multiple-attribute decision-making problem, proposed as the Cartesian product of the universal set X and the set Q. The goal of this work is to investigate this idea in a matrix setting by introducing the concept of complex Q-fuzzy matrices (CQFM). We discuss some new elementary operations, including the complement of CQFM, the union, and the intersection between two CQFMs. Moreover, we will show important algebraic properties that link these processes within a theoretical framework and provide numerical examples based on CQFM. Then, we promote techniques for determining and adjoining as well as develop methods for computing composition functions to determine the greatest and least eigenvalues of CQFMs. Finally, we will present a scenario for an administrative problem related to evaluating a chain of restaurants according to our proposed concept, in which the importance of the mathematical tools provided in this work will be demonstrated in addressing this scenario.

1. INTRODUCTION

Fuzzy set theory [1], introduced by Lotfizadeh in 1965, marked a significant shift in mathematics and decision science. By allowing elements to have degrees of belonging rather than binary containment, fuzzy set theory provided a powerful framework for modeling the uncertainty and ambiguity inherent in real-world problems. Over the decades, this fundamental concept has been widely generalized to address more complex forms of uncertainty. Uncertainty models have since undergone significant development to address highly complex real-world problems. Numerous extensions, including interval fuzzy sets [2], Pythagorean fuzzy sets [3], complex fuzzy sets [4], and Q-type fuzzy sets [5], have greatly enhanced fuzzy mathematics' ability to handle uncertain, imprecise, and multidimensional information. Each extension was developed to

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overcome specific limitations of earlier versions and to provide a richer mathematical framework for dealing with uncertainty in diverse scientific and engineering applications.These concepts formed an effective integration with other mathematical tools such as algebra [6, 7], topology [8, 9], statistics [10, 11], complex analysis [12], and others [13]- [16], which provided flexibility in dealing with everyday life problems in a more comprehensive way.

Among these developments, Q-fuzzy sets introduce an additional set of parameters (Q), enabling the evaluation of belonging values according to diverse criteria or contexts. These parameters greatly enhance the flexibility of fuzzy modeling in multi-attribute environments, making Q-fuzzy sets particularly useful for decision-making and addressing real-world problems. Simultaneously, complex fuzzy sets (CFSs) [4] expand the scope of traditional fuzzy sets by assigning belonging scores with complex values, where the amplitude describes the degree of belonging, while the periodic phase boundaries, temporal behavior, or hidden structural information are encoded. Consequently, complex fuzzy models have demonstrated superior performance in applications involving periodic phenomena, signal analysis, communication systems, image processing, and intelligent computing.

Matrix theory plays an essential role in mathematical design as an efficient representation of relationships among variables and system properties. Consequently, a great many fuzzy matrix models have been developed, including fuzzy matrices, intuitive fuzzy, complex fuzzy, and their various generalizations, where these matrix constructs have become indispensable tools in graph theory, optimization, artificial intelligence, pattern recognition, network analysis, and decision support systems due to their ability to represent uncertain relational data in a concise algebraic form. Figure 1 illustrates the cognitive tracing of the mathematical tools presented in this work, showing that these tools are an extension of previous algebraic concepts. Pal [17] explain the mechanism of employing fuzzy matrices in a number of areas of daily life. Prathab et al. [18] discussed the relationships between different types of fuzzy matrices. Fatma and Hassan [19] introduced a method of the Q-fuzzy matrix. Khan et al. [20] introduced the complex neutrosophic models in matrix form as an extension of complex fuzzy matrix. Al-Sharqi et al. [21, 22] stretch out the Q-F matrix to a new form called Q-neutrosophic matrix and check its applications.This matrix has gained approval and satisfaction from many researchers, prompting them to adopt it across many fields [23]- [29] and employ both established and innovative approaches.

Building on this research, we introduce a new algebraic concept, the CQ-F-matrix (CQ-FM), an extension of the previous work mentioned above that leverages matrix properties. These tools are powerful instruments for handling uncertain data by leveraging matrix properties, and we will apply them to solve some administrative problems related to the philosophy of administrative decision-making with clear characteristics.

The design of this article is presented as follows: to assist our discussion, we provide some background definitions of CQ-FSM in Part 2. In Part 3, we will provide a formal definition of a matrix system based on CQ-FM. Following this, we will present the algebraic operations and mathematical properties of this system and provide several examples illustrating how these tools work. Ultimately, we will use these tools to build a multi-step algorithm to solve an everyday decision-making problem. With Part 4. we will present a scenario for an administrative problem related to evaluating a chain of restaurants according to our proposed concept, in which the importance of the mathematical tools provided in this work will be demonstrated in addressing this scenario.Finally, we will provide a comprehensive comparison between the tools presented and the previous algebraic tools.

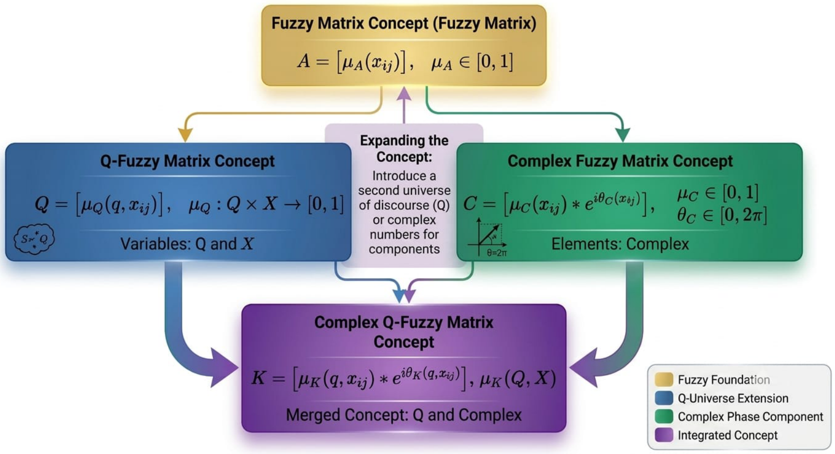

FIGURE 1. The historical sequence of previous studies is explained

2. PRELIMINARIES

In this part, we will reintroduce the formal definition of the concept of CQFS, which we will rely on to build the tools presented in this work.

Definition 2.1. *[26] A CQ–FS $\mathcal{R}_Q$ on a reference set X and a Q-set (X, Q) is given in the following form:*

$$\mathcal{P}_Q = \left\{ \langle x,\ \ddot{\mu}_{\mathcal{P}_Q}(x, q) \rangle \ :\ x \in X,\ q \in Q \right\},$$

where

$$\ddot{\mu}_{\mathcal{P}_Q} : (X \times Q) \longrightarrow \left\{ \ddot{a} : \ddot{a} \in \mathbb{C},\ |\ddot{a}| \leq 1 \right\}$$

is a complex TM value represented in polar form by

$$\ddot{\mu}_{\mathcal{P}_Q}(x, q) = \rho_{\mathcal{P}_Q}(x, q)\, e^{\,\mathrm{i}2\pi\varphi_{\mathcal{P}_Q}(x,q)},$$

such that

$$\rho_{\mathcal{P}_Q}(x, q) \in [0, 1] \qquad \text{and} \qquad \varphi_{\mathcal{P}_Q}(x, q) \in [0, 1],$$

so that the argument $2\pi\varphi_{\mathcal{P}_Q}(x, q)$ lies in $[0, 2\pi]$.

3. MATRIX THEORY FOR COMPLEX Q-FUZZY SETS

Definition 3.1. *Consider a reference set $X = \{x_1, x_2, x_3, \ldots, x_n\}$ and a set $Q = \{q_1, q_2, q_3, \ldots, q_m\}$ such that $X \times Q$ is the product between both of them and A_Q be CQ-FS over $X \times Q$. Then, the CQ-FS Y_Q given in matrix form as following:*

$$A_{Q_{n\times m}} = [a_{Q_{i\times j}}]_{n\times m} = \begin{cases} |\ddot{\mu}_{\mathcal{P}_Q}(x, q)|, & \text{if } (x, q) \in X \times Q, \\ (0), & \text{if } (x, q) \notin X \times Q. \end{cases}$$

Where $\ddot{\mu}_{\mathcal{P}_Q}(x, q) = \rho_{\mathcal{P}_Q}(x, q) \cdot e^{i2\pi(\varphi_{\mathcal{P}_Q}(x,q))}$ represents degrees of complex membership of truth on (x, q). Throughout this paper, we will utilize the abbreviation CQ-FSM$_{n\times m}$ for complex Q-fuzzy matrix over $X \times Q$.

Example 3.2. *Assume that $X = \{x_1, x_2\}$ represent two cars under consideration according to the following criteria, represented by $Q = \{q_1, q_2, q_3\}$ where q_1 = Engine power, q_2 = Color and q_3 = Price. Then the following Y_Q describes the "attractiveness of cars" that is considered for purchase:*

$$A_Q = \{\langle \frac{0.2e^{2\pi i(0,4)}}{(x_1, q_1)}, \frac{0.5e^{2\pi i(0,7)}}{(x_1, q_2)}, \frac{0.1e^{2\pi i(0,2)}}{(x_1, q_3)}, \frac{0.6e^{2\pi i(0,8)}}{(x_2, q_1)}, \frac{0.3e^{2\pi i(0,9)}}{(x_2, q_2)}, \frac{0.6e^{2\pi i(0,6)}}{(x_2, q_3)} \rangle\}$$

Where,

$$\begin{aligned}
0.2e^{2\pi i(0,4)} &= 0.2(\cos(0.8\pi) + i\sin(0.8\pi)) = 0.2(-0.80 + i0.58) = (-0.16 + i0.116)\\
|-0.16 + i0.116| &= \sqrt{0.0256 + 0.0134} = 0.1974\\
0.5e^{2\pi i(0,7)} &= 0.5(\cos(1.4\pi) + i\sin(1.4\pi)) = 0.5(-0.30 - i0.95) = (-0.154 - i0.475)\\
|-0.154 - i0.475| &= \sqrt{0.0237 + 0.2256} = 0.4993\\
0.1e^{2\pi i(0,2)} &= 0.1(\cos(0.4\pi) + i\sin(0.4\pi)) = 0.1(0.309 + i0.951) = (0.0309 + i0.0951)\\
|0.0309 + i0.0951| &= \sqrt{0.00095 + 0.00904} = 0.0999\\
0.6e^{2\pi i(0,8)} &= 0.6(\cos(1.6\pi) + i\sin(1.6\pi)) = 0.6(0.309 - i0.951) = (0.1854 - i0.5706)\\
|0.1854 - i0.5706| &= \sqrt{0.0343 + 0.3255} = 0.5999\\
0.3e^{2\pi i(0,9)} &= 0.3(\cos(1.8\pi) + i\sin(1.8\pi)) = 0.3(0.8090 - i0.5877) = (0.2427 - i0.1763)\\
|0.2427 - i0.1763| &= \sqrt{0.0589 + 0.0310} = 0.2998\\
0.6e^{2\pi i(0,6)} &= 0.6(\cos(1.2\pi) + i\sin(1.2\pi)) = 0.6(-0.8090 - i0.5877) = (-0.4854 - i0.3526)\\
|-0.4854 - i0.3526| &= \sqrt{0.2356 + 0.1243} = 0.5999
\end{aligned}$$

Now, based on the above-mentioned CQFS values, the CQFS is given in matrix form as follows:

$$A_{Q_{3\times 2}} = \begin{bmatrix} & q_1 & q_2 & q_3 \\ x_1 & 0.1974 & 0.4993 & 0.0999 \\ x_2 & 0.5999 & 0.2998 & 0.5999 \end{bmatrix}$$

Definition 3.3. *Assume that $A_{Q_{n\times m}}$ is CQFM. Then this matrix is called zero CQFM if $[a_{Q_{i\times j}}]_{n\times m} = [0]$ for all i and j and this matrix is denoted by $0_{Q_{i\times j}}$.*

Definition 3.4. *Assume that $A_{Q_{n\times m}}$ is CQFM. Then this matrix is called universal CQFM if $[a_{Q_{i\times j}}]_{n\times m} = [1]$ for all i and j and this matrix is denoted by $1_{Q_{i\times j}}$.*

Example 3.5. *Let*

$$0_{Q_{3\times 2}} = \begin{bmatrix} & q_1 & q_2 & q_3 \\ x_1 & (0) & 0 & 0 \\ x_2 & 0 & 0 & 0 \end{bmatrix}$$

and

$$1_{Q_{3\times 2}} = \begin{bmatrix} & q_1 & q_2 & q_3 \\ x_1 & (1) & 1 & 1 \\ x_2 & 1 & 1 & 1 \end{bmatrix}$$

Both matrices above are called the zero CQFM matrix and the universal CQFM matrix, respectively.

Definition 3.6. *For two CQFMs $A_{Q_{n\times m}}$ and $B_{Q_{n\times m}}$ defined on the product between X and Q. Then $A_{Q_{n\times m}}$ is CQF-submatrix of $B_{Q_{n\times m}}$ and is denoted by $[A_{Q_{n\times m}}] \sqsubseteq [B_{Q_{n\times m}}]$ if $a_{Q_{i\times j}} \leq b_{Q_{i\times j}}$ for all i and j.*

Definition 3.7. *For two CQFMs $A_{Q_{n\times m}}$ and $B_{Q_{n\times m}}$ defined on the product between X and Q. Then $A_{Q_{n\times m}}$ is CQF-equal matrix of $B_{Q_{n\times m}}$ and is denoted by $[A_{Q_{n\times m}}] = [B_{Q_{n\times m}}]$ if $a_{Q_{i\times j}} = b_{Q_{i\times j}}$ for all i and j.*

Definition 3.8. *Assume that $A_{Q_{n\times m}}$ is CQFM. Then this matrix is called sequel universal CQFM if the number of rows (n) equals the number of columns (m) for all i and j.*

Definition 3.9. *For a CQFM $A_{Q_{n\times m}}$ defined on the product between X and Q. Then the complement of $A_{Q_{n\times m}}$ is denoted by $[A^C_{Q_{n\times m}}]$ if $c_{Q_{i\times j}} = 1 - a_{Q_{i\times j}}$ for all i and j.*

Definition 3.10. *For two CQFMs $A_{Q_{n\times m}}$ and $B_{Q_{n\times m}}$ defined on the product between X and Q. Then the union between $A_{Q_{n\times m}}$ and $B_{Q_{n\times m}}$ is $U_{Q_{n\times m}}$ and is denoted by $[A_{Q_{n\times m}}] \sqcup [B_{Q_{n\times m}}]$ if $u_{Q_{i\times j}} = [\max(a_{Q_{i\times j}}, b_{Q_{i\times j}})]$ for all i and j.*

Definition 3.11. *For two CQFMs $A_{Q_{n\times m}}$ and $B_{Q_{n\times m}}$ defined on the product between X and Q. Then the intersection between $A_{Q_{n\times m}}$ and $B_{Q_{n\times m}}$ is $S_{Q_{n\times m}}$ and is denoted by $[A_{Q_{n\times m}}] \sqcap [B_{Q_{n\times m}}]$ if $s_{Q_{i\times j}} = [\min(a_{Q_{i\times j}}, b_{Q_{i\times j}})]$ for all i and j.*

Example 3.12. *Assume that the following CQFMs are defined on the product between X and Q:*

$$A_{Q_{2\times 3}} = \begin{bmatrix} 0.36 & 0.52 & 0.12 \\ 0.48 & 0.73 & 0.59 \end{bmatrix}, \quad B_{Q_{2\times 3}} = \begin{bmatrix} 0.11 & 0.76 & 0.82 \\ 1 & 0 & 0.63 \end{bmatrix}$$

Then,

$$U_{Q_{2\times 3}} = \begin{bmatrix} 0.36 & 0.76 & 0.82 \\ 1 & 0.73 & 0.63 \end{bmatrix}, \quad S_{Q_{2\times 3}} = \begin{bmatrix} 0.11 & 0.52 & 0.12 \\ 0.48 & 0 & 0.59 \end{bmatrix}$$

and

$$A^C_{Q_{2\times 3}} = \begin{bmatrix} 0.64 & 0.48 & 0.18 \\ 0.52 & 0.27 & 0.41 \end{bmatrix}$$

Proposition 3.13. *Assume that a CQFM $A_{Q_{n\times m}}$ is defined on the product between X and Q. Then the following points are verified:*

(1) $((A_{Q_{n\times m}})^c)^c = A_{Q_{n\times m}}$.
(2) $((0_{Q_{n\times m}})^c)^c = 1_{Q_{n\times m}}$.

Proof. Follows from Definition 3.9. □

Proposition 3.14. *For three CQFMs $A_{Q_{n\times m}}$, $B_{Q_{n\times m}}$ and $D_{Q_{n\times m}}$ defined on the product between X and Q. Then the following points are verified:*

(1) *If $A_{Q_{n\times m}} \leq B_{Q_{n\times m}}$ and $B_{Q_{n\times m}} \leq D_{Q_{n\times m}}$ then $A_{Q_{n\times m}} \leq D_{Q_{n\times m}}$.*
(2) *If $A_{Q_{n\times m}} = B_{Q_{n\times m}}$ and $B_{Q_{n\times m}} = D_{Q_{n\times m}}$ then $A_{Q_{n\times m}} = D_{Q_{n\times m}}$.*

Proposition 3.15. *For three CQFMs $A_{Q_{n\times m}}$, $B_{Q_{n\times m}}$ and $D_{Q_{n\times m}}$ defined on the product between X and Q. Then the following points are verified:*

(1) $A_{Q_{n\times m}} \sqcup B_{Q_{n\times m}} = B_{Q_{n\times m}} \sqcup A_{Q_{n\times m}}$.
(2) $A_{Q_{n\times m}} \sqcap B_{Q_{n\times m}} = B_{Q_{n\times m}} \sqcap A_{Q_{n\times m}}$.
(3) $(A_{Q_{n\times m}} \sqcup B_{Q_{n\times m}}) \sqcup D_{Q_{n\times m}} = A_{Q_{n\times m}} \sqcup (B_{Q_{n\times m}} \sqcup D_{Q_{n\times m}})$.
(4) $(A_{Q_{n\times m}} \sqcap B_{Q_{n\times m}}) \sqcap D_{Q_{n\times m}} = A_{Q_{n\times m}} \sqcap (B_{Q_{n\times m}} \sqcap D_{Q_{n\times m}})$.
(5) $A_{Q_{n\times m}} \sqcup (B_{Q_{n\times m}} \sqcap D_{Q_{n\times m}}) = (A_{Q_{n\times m}} \sqcup B_{Q_{n\times m}}) \sqcap (A_{Q_{n\times m}} \sqcup D_{Q_{n\times m}})$.
(6) $A_{Q_{n\times m}} \sqcap (B_{Q_{n\times m}} \sqcup D_{Q_{n\times m}}) = (A_{Q_{n\times m}} \sqcap B_{Q_{n\times m}}) \sqcup (A_{Q_{n\times m}} \sqcap D_{Q_{n\times m}})$.

Proof. (i). Take

$$A_{Q_{n\times m}} \sqcup B_{Q_{n\times m}} = \max(a_{Q_{n\times m}}, b_{Q_{n\times m}}) = \max(b_{Q_{n\times m}}, a_{Q_{n\times m}}) = B_{Q_{n\times m}} \sqcup A_{Q_{n\times m}}.$$

(ii). Take

$$A_{Q_{n\times m}} \sqcap B_{Q_{n\times m}} = \min(a_{Q_{n\times m}}, b_{Q_{n\times m}}) = \min(b_{Q_{n\times m}}, a_{Q_{n\times m}}) = B_{Q_{n\times m}} \sqcap A_{Q_{n\times m}}.$$

(iii).

$$\begin{aligned}(A_{Q_{n\times m}} \sqcup B_{Q_{n\times m}}) \sqcup D_{Q_{n\times m}} &= \max(\max(A_{Q_{n\times m}}, B_{Q_{n\times m}}), D_{Q_{n\times m}})\\ &= \max(A_{Q_{n\times m}}, \max(B_{Q_{n\times m}}, D_{Q_{n\times m}}))\\ &= \max(A_{Q_{n\times m}}, (B_{Q_{n\times m}} \sqcup D_{Q_{n\times m}}))\\ &= A_{Q_{n\times m}} \sqcup (B_{Q_{n\times m}} \sqcup D_{Q_{n\times m}}).\end{aligned}$$

(iv).

$$\begin{aligned}(A_{Q_{n\times m}} \sqcap B_{Q_{n\times m}}) \sqcap D_{Q_{n\times m}} &= \min(\min(A_{Q_{n\times m}}, B_{Q_{n\times m}}), D_{Q_{n\times m}})\\ &= \min(A_{Q_{n\times m}}, \min(B_{Q_{n\times m}}, D_{Q_{n\times m}}))\\ &= \min(A_{Q_{n\times m}}, (B_{Q_{n\times m}} \sqcap D_{Q_{n\times m}}))\\ &= A_{Q_{n\times m}} \sqcap (B_{Q_{n\times m}} \sqcap D_{Q_{n\times m}}).\end{aligned}$$

□

Definition 3.16. *For two CQFMs $A_{Q_{n\times m}}$ and $B_{Q_{n\times m}}$ defined on the product between X and Q. Then the addition $(+)$ between $A_{Q_{n\times m}}$ and $B_{Q_{n\times m}}$ is $S_{Q_{n\times m}}$ and is denoted by $[A_{Q_{n\times m}}] + [B_{Q_{n\times m}}]$ for all i and j and is given as following:*

$$[A_{Q_{n\times m}}] + [B_{Q_{n\times m}}] = \left[\frac{a_{Q_{n\times m}} + b_{Q_{n\times m}}}{2}\right]$$

Definition 3.17. *For two CQFMs $A_{Q_{n\times m}}$ and $B_{Q_{n\times m}}$ defined on the product between X and Q. Then the subtraction $(-)$ between $A_{Q_{n\times m}}$ and $B_{Q_{n\times m}}$ is $S_{Q_{n\times m}}$ and is denoted by $[A_{Q_{n\times m}}] - [B_{Q_{n\times m}}]$ for all i and j and is given as following:*

$$[A_{Q_{n\times m}}] - [B_{Q_{n\times m}}] = [|a_{Q_{n\times m}} - b_{Q_{n\times m}}|]$$

Definition 3.18. *For two CQFMs $A_{Q_{n\times m}}$ and $B_{Q_{n\times m}}$ defined on the product between X and Q. Then the multiplication $(\times)$ between $A_{Q_{n\times m}}$ and $B_{Q_{n\times m}}$ is $S_{Q_{n\times m}}$ and is denoted by $[A_{Q_{n\times m}}] \times [B_{Q_{n\times m}}]$ for all i and j and is given as following:*

$$[A_{Q_{n\times m}}] \times [B_{Q_{n\times m}}] = [|a_{Q_{n\times m}} \times b_{Q_{n\times m}}|]$$

Definition 3.19. *For CQFM $A_{Q_{n\times m}}$ defined on the product between X and Q. Then the scalar multiplication $k \in [0,1]$ of $A_{Q_{n\times m}}$ is $S_{Q_{n\times m}}$ and is denoted by $(k[A_{Q_{n\times m}}])$ for all i and j and is given as following:*

$$k[A_{Q_{n\times m}}] = [|ka_{Q_{n\times m}}|]$$

Definition 3.20. *For two CQFMs $A_{Q_{n\times m}}$ and $B_{Q_{n\times m}}$ defined on the product between X and Q. Then the maximum operation between $A_{Q_{n\times m}}$ and $B_{Q_{n\times m}}$ is $S_{Q_{n\times m}}$ and is denoted by $[A_{Q_{n\times m}}] \vee [B_{Q_{n\times m}}]$ for all i and j and is given as following:*

$$S_{Q_{n\times m}} = [A_{Q_{n\times m}}] \vee [B_{Q_{n\times m}}] = [\max(a_{Q_{n\times m}} \times b_{Q_{n\times m}})]$$

Definition 3.21. *For two CQFMs $A_{Q_{n\times m}}$ and $B_{Q_{n\times m}}$ defined on the product between X and Q. Then the minimum operation between $A_{Q_{n\times m}}$ and $B_{Q_{n\times m}}$ is $S_{Q_{n\times m}}$ and is denoted by $[A_{Q_{n\times m}}] \wedge [B_{Q_{n\times m}}]$ for all i and j and is given as following:*

$$S_{Q_{n\times m}} = [A_{Q_{n\times m}}] \wedge [B_{Q_{n\times m}}] = [\min(a_{Q_{n\times m}} \times b_{Q_{n\times m}})]$$

4. Employment complex Q-fuzzy matrices for administrative decision-making

This section of the article will highlight the importance of the tools provided in addressing administrative challenges. Senior management in restaurant associations generally faces increasing pressure to make diverse strategic decisions regarding restaurant evaluations within the workplace. This is due to the incomplete nature of the data, the conflicting opinions of the evaluation committee, and the presence of experts who may not offer unbiased perspectives. Furthermore, the currently available removable data is largely too broad, both in reality and in practice, for models to encompass.

In the following scenario, we will assume that a two-member committee (X_1,X_2) is formed to evaluate the performance of four restaurants operating in a tourist city. These restaurants are sets into set $\mathbb{Y} = \{y_1, y_2, y_3, y_4\}$, and the criteria the committee uses are also grouped into set $\mathbb{Q} = \{q_1, q_2, q_3\}$, which are, respectively: The variety of food, the restaurant's aesthetics, and the popularity of tourism.

After conducting a comprehensive survey of these restaurants by the residents, we obtain the two matrices below, which represent the opinions of each of the residents based on the above criteria.

$$X_1 = \begin{array}{cc} {}^{Y}/_{Q} & \begin{matrix} q_1 & q_2 & q_3 \end{matrix} \\ \begin{matrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{matrix} & \begin{bmatrix} .4 & .6 & .3 \\ .8 & .1 & .5 \\ .2 & .4 & .6 \\ .5 & .5 & .9 \end{bmatrix} \end{array}$$

$$X_2 = \begin{array}{cc} {}^{Y}/_{Q} & \begin{matrix} q_1 & q_2 & q_3 \end{matrix} \\ \begin{matrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{matrix} & \begin{bmatrix} .5 & .8 & .6 \\ .9 & .6 & .9 \\ .3 & .2 & .1 \\ .4 & 5 & .7 \end{bmatrix} \end{array}$$

To deal with this data generated from the opinions of the residents in the two matrices above, we will resort to generating the following algorithm.

<u>Algorithm:</u>

To implement this performance in accordance with the mathematical structure of our proposed concept, we will introduce the following algorithm.

Step 1. Merge the two matrices, which represent the residents opinions, into a single matrix using one of the mathematical operations.

Step 2. Using a comparison matrix means that we will compare the numerical value of each opinion about one of the restaurants with other restaurants according to the specified standard.

Step 3. Sort the results in descending order according to the values obtained in step2.

Step 4. End Algorithm.

Now we will implement this algorithm to deal with the above issue, where first we will execute step 1. Therefore, we obtain the matrix X_3 by adding the two matrices.

$$X_2 = \begin{array}{cc} {}^{Y}/_{Q} & \begin{matrix} q_1 & q_2 & q_3 \end{matrix} \\ \begin{matrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{matrix} & \begin{bmatrix} .45 & .7 & .45 \\ .85 & .35 & .7 \\ .25 & .3 & .35 \\ .45 & .5 & .8 \end{bmatrix} \end{array}$$

Then, by using the comparison between the numerical values, we obtain the comparison matrix C as follows:

$$C = \begin{bmatrix} {}^{Y}/_{Q} & q_1 & q_2 & q_3 \\ y_1 & 1 & 3 & 1 \\ y_2 & 3 & 1 & 2 \\ y_3 & 0 & 0 & 0 \\ y_4 & 2 & 2 & 3 \end{bmatrix}$$

Finally, by ranking the numerical values for each restaurant, we obtain $y_4 > y_2 > y_1 > y_3$. Therefore, the restaurant y_4 is the best-rated of the four restaurants.

5. Comparison between Fuzzy Matrix, Q-Fuzzy Matrix, Complex Fuzzy Matrix, and Complex Q-Fuzzy Matrix

Matrix-based fuzzy models have become indispensable tools for representing uncertain relationships among objects in various fields, including decision-making, graph theory, network analysis, optimization, image processing, and artificial intelligence. Over the years, several fuzzy matrix models have been proposed to improve the expressive capability of classical fuzzy matrices by incorporating additional sources of uncertainty. Among these developments, Fuzzy Matrices, Q-Fuzzy Matrices, Complex Fuzzy Matrices, and the newly introduced Complex Q-Fuzzy Matrices represent successive stages in the evolution of fuzzy matrix theory. A Fuzzy Matrix (FM) is the most fundamental matrix representation in fuzzy mathematics. Each entry of a fuzzy matrix is a real-valued membership degree belonging to the interval ([0,1]), indicating the strength of the relationship between two objects. Owing to its simple structure, the fuzzy matrix has been extensively employed in relational modeling, clustering, pattern recognition, and decision support systems. However, its representation is limited to a single membership value and therefore cannot adequately describe uncertainty arising from multiple parameters or dynamic phenomena.

To address this limitation, the Q-Fuzzy Matrix (QFM) extends the classical fuzzy matrix by associating each matrix entry with an additional parameter set (Q). Instead of assigning a single membership value to each matrix element, the membership function is expressed as $\mu_A(x_{ij}, q), \qquad q \in Q$, allowing the relationship between two objects to be evaluated under different criteria, conditions, or contexts. This parameterized representation significantly increases modeling flexibility and makes Q-fuzzy matrices suitable for multi-criteria decision-making, knowledge representation, and systems where uncertainty varies according to external parameters. Nevertheless, the membership values remain real-valued and therefore cannot capture periodic or phase-dependent information.

The Complex Fuzzy Matrix (CFM) generalizes the fuzzy matrix from another perspective by replacing each real-valued membership degree with a complex-valued membership. Each matrix entry is commonly represented as $\mu_A(x_{ij}) = r_{ij}e^{i\theta_{ij}}$,

where $(r_{ij} \in [0,1])$ denotes the amplitude and (θ_{ij}) represents the phase component. While the amplitude measures the degree of membership, the phase conveys additional information related to periodicity, temporal variation, oscillatory behavior, or hidden structural characteristics. Consequently, complex fuzzy matrices provide a richer mathematical representation for applications such as signal processing, communication systems, image analysis, and dynamic network modeling. However, this model does not explicitly account for parameter-dependent uncertainty.

The Complex Q-Fuzzy Matrix (CQFM) integrates the advantages of both Q-fuzzy matrices and complex fuzzy matrices into a unified algebraic framework. In this model, every matrix entry is represented by a parameterized complex-valued membership function of the form

$\mu_A(x_{ij}, q) = r_{ij}(q)e^{i\theta_{ij}(q)}, \qquad q \in Q$. Accordingly, each entry simultaneously incorporates two complementary dimensions of information. The amplitude describes the degree of membership under a specific parameter, whereas the phase characterizes dynamic, cyclic, or temporal behavior associated with that parameter. This dual representation enables Complex Q-fuzzy matrices to model parameter-dependent uncertainty and phase-sensitive information simultaneously, thereby substantially enhancing the descriptive capability of conventional fuzzy matrix models. From a theoretical perspective, the progression from fuzzy matrices to Complex Q-fuzzy matrices reflects a continuous enhancement of uncertainty representation. Classical fuzzy matrices describe only the intensity of relationships. Q-fuzzy matrices extend this representation by introducing parameterization, allowing the same relationship to be evaluated under multiple conditions. Complex fuzzy matrices enrich the membership representation through complex-valued grades that capture both intensity and phase information. Finally, Complex Q-fuzzy matrices combine these two extensions into a single mathematical framework, offering a comprehensive representation capable of modeling multidimensional, parameter-dependent, and dynamic uncertainty.

Despite the considerable development of fuzzy matrix theory, an important research gap still exists. Existing studies have investigated Q-fuzzy matrices and complex fuzzy matrices independently, with each model addressing only one aspect of uncertainty. Q-fuzzy matrices successfully represent parameter-dependent uncertainty but ignore phase information, whereas complex fuzzy matrices capture phase-dependent characteristics but do not accommodate multiple parameters. To the best of our knowledge, a unified matrix framework capable of simultaneously representing both parameterization and complex-valued membership information has not been systematically established. The proposed Complex Q-Fuzzy Matrix is introduced to bridge this gap by integrating the strengths of both models into a single coherent algebraic structure. This unified framework not only enriches fuzzy matrix theory but also provides a powerful mathematical tool for modeling complex uncertain systems encountered in intelligent decision-making, machine learning, communication networks, image processing, and advanced information systems.For further comparison, we present the following diagram which shows the fundamental difference in the structural frameworks of the four concepts.

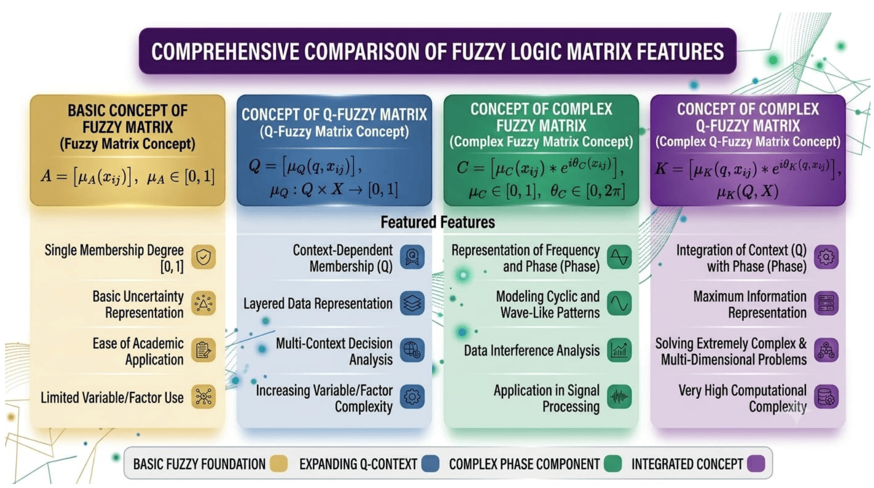

FIGURE 2. A comparison between the tools previously used and the tools presented in this work.

6. CONCLUSION

Following the research works in the fuzzy environment mentioned in the literature section,in this paper, we investigated this idea in a matrix setting by introducing the concept of complex Q-fuzzy matrices

(CQFM). We discussed some new elementary operations, including the complement of CQFM, the union, and the intersection between two CQFMs. Moreover, we demonstrated important algebraic properties that link these processes within a theoretical framework, and we introduced numerical examples based on CQFM. Then, we promoted techniques for determining and adjoining as well as developed methods for computing composition functions to determine the greatest and least eigenvalues of CQFMs. Finally, we presented a scenario for an administrative problem related to evaluating a chain of restaurants according to our proposed concept, in which the importance of the mathematical tools provided in this work will be demonstrated in addressing this scenario.

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References

[1] L.A. Zadeh, Fuzzy Sets, Inf. Control 8 (1965), 338–353. https://doi.org/10.1016/s0019-9958(65)90241-x.
[2] M. Akram, S.M.U. Shah, M.M.A. Al-Shamiri, S.A. Edalatpanah, Fractional Transportation Problem under Interval-Valued Fermatean Fuzzy Sets, AIMS Math. 7 (2022), 17327–17348. https://doi.org/10.3934/math.2022954.
[3] M.C. Bozyigit, M. Olgun, M. Ünver, Circular Pythagorean Fuzzy Sets and Applications to Multi-Criteria Decision Making, Informatica 34 (2023), 713–742. https://doi.org/10.15388/23-infor529.
[4] D. Ramot, R. Milo, M. Friedman, A. Kandel, Complex Fuzzy Sets, IEEE Trans. Fuzzy Syst. 10 (2002), 171–186. https://doi.org/10.1109/91.995119.
[5] F. Adam, N. Hassan, Q-Fuzzy Soft Set, Appl. Math. Sci. 8 (2014), 8689–8695. https://doi.org/10.12988/ams.2014.410865.
[6] K. Suayngam, R. Prasertpong, W. Nakkhasen, P. Julatha, A. Iampan, Pythagorean Fuzzy Sets: A New Perspective on IUP-Algebras, Int. J. Innov. Comput. Inf. Control 21 (2025), 339–357. https://doi.org/10.24507/ijicic.21.02.339.
[7] A.A. Hazaymeh, A. Bataihah, Neutrosophic Fuzzy Metric Spaces and Fixed Points for Contractions of Nonlinear Type, Neutrosoph. Sets Syst. 77 (2025), 96–112.
[8] O. Dalkılıç, Relational Assessment in Hypersoft Sets: a Decision-Maker Driven Approach, Comput. Appl. Math. 45 (2026), 288. https://doi.org/10.1007/s40314-026-03674-9.
[9] N. Sabry, F. Mohammed, Betam-Closed Sets in Fuzzy Neutrosophic Topological Spaces, Int. J. Neutrosophic Sci. 23 (2024), 160–169. https://doi.org/10.54216/ijns.230412.
[10] T.M. Ali, F.M. Mohammed, Some Perfectly Continuous Functions via Fuzzy Neutrosophic Topological Spaces, Int. J. Neutrosophic Sci. 18 (2022), 147–182. https://doi.org/10.54216/ijns.180416.
[11] D. Pratama, B. Yusoff, J.C.R. Alcantud, L. Abdullah, F. Al-Sharqi, Improved Ordering and Aggregation Operators for Circular Intuitionistic Fuzzy Sets in Multi-Criteria Decision-Making, Complex Intell. Syst. 12 (2026), 205. https://doi.org/10.1007/s40747-026-02320-5.
[12] Y. Al-Qudah, F. Al-Sharqi, M. Mishlish, M.M. Rasheed, Hybrid Integrated Decision-Making Algorithm Based on AO of Possibility Interval-Valued Neutrosophic Soft Settings, Int. J. Neutrosophic Sci. 22 (2023), 84–98. https://doi.org/10.54216/ijns.220306.
[13] Y. Al-Qudah, K. Alhazaymeh, N. Hassan, H. Qoqazeh, M. Almousa, et al., Transitive Closure of Vague Soft Set Relations and Its Operators, Int. J. Fuzzy Log. Intell. Syst. 22 (2022), 59–68. https://doi.org/10.5391/ijfis.2022.22.1.59.
[14] M.U. Romdhini, F. Al-Sharqi, R.H. Al-Obaidi, Z.M. Rodzi, Modeling Uncertainties Associated with Decision-Making Algorithms Based on Similarity Measures of Possibility Belief Interval-Valued Fuzzy Hypersoft Setting, Neutrosoph. Sets Syst. 77 (2025), 282–309.
[15] Z.M. Rodzi, N.A.M. Rosly, N.A.M. Zaik, M.H. Rusli, G. Ahmad, et al., A DEMATEL Analysis of the Complex Barriers Hindering Digitalization Technology Adoption in the Malaysia Agriculture Sector, J. Intell. Syst. Internet Things 13 (2024), 21–30. https://doi.org/10.54216/jisiot.130102.
[16] A. Bataihah, A. Hazaymeh, Quasi Contractions and Fixed Point Theorems in the Context of Neutrosophic Fuzzy Metric Spaces, Eur. J. Pure Appl. Math. 18 (2025), 5785. https://doi.org/10.29020/nybg.ejpam.v18i1.5785.
[17] M. Pal, Fuzzy Matrices, in: Recent Developments of Fuzzy Matrix Theory and Applications, Springer Nature Switzerland, Cham, pp. 1–63, (2024). https://doi.org/10.1007/978-3-031-56936-4_1.
[18] H. Prathab, N. Ramalingam, E. Janaki, A. Bobin, V. Kamalakannan, et al., Interval Valued Secondary k-Range Symmetric Fuzzy Matrices with Generalized Inverses, IAENG Int. J. Comput. Sci. 51 (2024), 2051–2066.
[19] F. Adam, N. Hassan, Q-Fuzzy Soft Matrix and Its Application, in: AIP Conference Proceedings, AIP Publishing LLC, 2014, pp. 772–778. https://doi.org/10.1063/1.4882573.

[20] M. Khan, S. Anis, K. Bibi, S. Iqbal, F. Smarandache, Complex Neutrosophic Soft Matrices Framework: An Application in Signal Processing, J. Math. 2021 (2021), 1–10. https://doi.org/10.1155/2021/8887824.

[21] F. Al-Sharqi, M.U. Romdhini, A. Al-Quran, Group Decision-Making Based on Aggregation Operator and Score Function of Q-Neutrosophic Soft Matrix, J. Intell. Fuzzy Syst. 45 (2023), 305–321. https://doi.org/10.3233/jifs-224552.

[22] Y. Al-Qudah, A.O. Hamadameen, N.A. Kh, F. Al-Sharqi, A New Generalization of Interval-Valued Q-Neutrosophic Soft Matrix and Its Applications, Int. J. Neutrosophic Sci. 25 (2025), 242–257. https://doi.org/10.54216/IJNS.250322.

[23] M.U. Romdhini, A. Al-Quran, F. Al-Sharqi, M.K. Tahat, A. Lutfi, Exploring the Algebraic Structures of Q-Complex Neutrosophic Soft Fields, Int. J. Neutrosophic Sci. 22 (2023), 93–105. https://doi.org/10.54216/ijns.220408.

[24] A. Hazaymeh, Y. Al-Qudah, F. Al-Sharqi, A. Bataihah, A Novel Q-Neutrosophic Soft under Interval Matrix Setting and Its Applications, Int. J. Neutrosophic Sci. 25 (2025), 156–168. https://doi.org/10.54216/ijns.250413.

[25] A. Malkawi, A. Rabaiah, Fuzzy Metric Spaces: Theory and Applications in Medical Imaging, Transportation, and Social Networks, Nonlinear Funct. Anal. Appl. 30 (2025), 1255–1272.

[26] M.U. Romdhini, Y. Al-Qudah, F. Al-Sharqi, A. Abdurahim, S.J. Shoja, An Innovative Approach to Complex Q-Fuzzy Set and Their Applications in Medical Diagnosis, Asia Pac. J. Math. 13 (2026), 110. https://doi.org/10.28924/apjm/13-110.

[27] M.U. Romdhini, A. Nawawi, F. Al-Sharqi, A. Al-Quran, S.R. Kamali, Wiener-Hosoya Energy of Non-Commuting Graph for Dihedral Groups, Asia Pac. J. Math. 11 (2024), 9. https://doi.org/10.28924/apjm/11-9.

[28] B. Hu, L. Bi, S. Dai, The Orthogonality between Complex Fuzzy Sets and Its Application to Signal Detection, Symmetry 9 (2017), 175. https://doi.org/10.3390/sym9090175.

[29] M.H. Mateen, T. Alsuraiheed, M. Hmissi, A New Algebraic Structure of Complex Pythagorean Fuzzy Subfield, IEEE Access 11 (2023), 145636–145650. https://doi.org/10.1109/access.2023.3346188.