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SPLIT DOMINATION NUMBER IN EDGE SEMI-MIDDLE GRAPH

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ABSTRACT. Let $G = (p, q)$ be a connected graph and $M_e(G)$ be its corresponding edge semi-middle graph. A dominating set $D \subseteq V[M_e(G)]$ is split dominating set $\langle V[M_e(G)] - D\rangle$ is disconnected. The minimum size of D is called the split domination number of $M_e(G)$ and is denoted by $\gamma_s[M_e(G)]$. In this paper we obtain several results on split domination number.

1. INTRODUCTION

Domination is an area in graph theory with an extensive research activity. We consider simple, finite, undirected, non-trivial and connected graphs for our study. In literature, the concept of graph theory terminology not presented here can be found in [1]. In a graph G, a set $D \subseteq V$ is dominating set of G if every vertex in $V - D$ is adjacent to some vertex in D. The domination number of a graph G is the minimum size of D. Some studies on domination in graphs were found in the papers [2–4, 6–15, 18–30]. The edge semi middle graph $M_e(G)$ of a graph G was studied [17] and is defined as follows. Let $V(G)$, $E(G)$ and $R(G)$ be the vertex set, edge set and regions set respectively. The edge semi-middle graph of a graph G, denoted by $M_e(G)$ is a graph whose vertex set is $V(G) \cup E(G) \cup R(G)$. The vertices of $M_e(G)$ are adjacent if and only if they corresponds to two adjacent edges of G or one corresponds to a vertex and other to an edge incident with it or one corresponds to edge and other to a region in which edge lie on the region.

Let $R' = \{r'_1, r'_2, ...r'_m\} \subseteq V[M_e(G)]$ for the region set $\{r_1, r_2, ...r_m\}$ of G. Let $V' = \{v'_1, v'_2, ...v'_p\} \subseteq V[M_e(G)]$ for the vertex set $\{v_1, v_2, ...v_p\}$ of G. Let $E' = \{e'_1, e'_2, ...e'_q\} \subseteq V[M_e(G)]$ for the edge set $\{e_1, e_2, ...e_q\}$ of G. The study of some domination parameters on jump graph [16] motivated us to introduce split domination number in edge semi middle graph.

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2. PRELIMINARIES

Theorem 2.1. *[5] For any graph G, $\gamma(G) \geq \lceil \frac{p}{1+\triangle(G)} \rceil$.*

Theorem 2.2. *[31] For the path P_n, $\gamma[M_e(P_n)] = \lceil \frac{n}{2} \rceil$.*

Theorem 2.3. *[31] For the cycle C_n, $\gamma[M_e(C_n)] = \lceil \frac{n}{2} \rceil$.*

Theorem 2.4. *[31] For any graph $G(p, q)$, $\gamma[M_e(G)] \geq \lceil \frac{p}{1+\triangle(G)} \rceil$.*

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3. Split Domination Number in Edge Semi-Middle Graph

A dominating set D of $M_e(G)$ is a split dominating set if $\langle V[M_e(G)] - D\rangle$ is disconnected. The minimum cardinality of D is called split domination number of $M_e(G)$ and is denoted by $\gamma_s[M_e(G)]$. A minimum split dominating set is denoted by $\gamma_s - set$.

In the Figure 1, the split dominating set of $M_e(G)$ is $D_4 = \{e_1^{'}, e_3^{'}, e_5^{'}, e_6^{'}\}, \gamma_s[M_e(G)] = 4$;

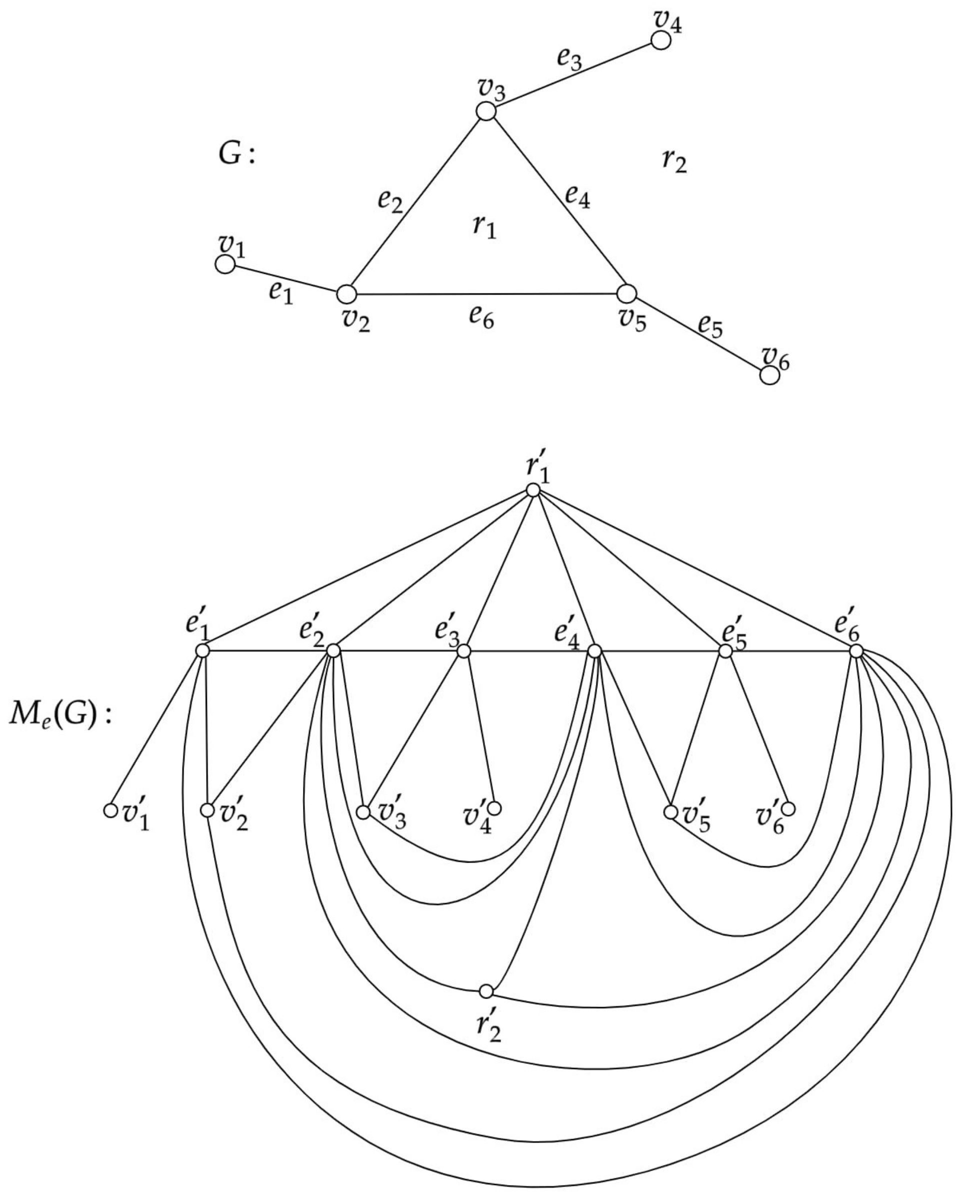

FIGURE 1. The graph G and its $M_e(G)$

We begin with the following observations.

Observation 3.1. *For every star $K_{1,n}, \gamma[M_e(K_{1,n})] = \gamma_s[M_e(K_{1,n})] = n$.*

Observation 3.2. *Let G be a tree, $\gamma_s[M_e(T)] = \gamma[M_e(T)]$.*

4. Results

Theorem 4.1. *For the path* $P_n, \gamma_s[M_e(P_n)] = \lceil \frac{n}{2} \rceil$.

Proof. Consider $G = P_n$. Let D be the dominating set of $M_e(G)$ and is defined as follows.

$$D = \begin{cases} e_1^{'}, e_3^{'}, ...e_{n-1}^{'} & \text{if n=2k.} \\ \\ e_1^{'}, e_3^{'}, ...e_{n-2}^{'}v_n^{'} & \text{if n=2k+1.} \end{cases}$$

Clearly, $V[M_e(P_n)] - D$ is disconnected. Thus $\gamma_s[M_e(P_n)] = \lceil \frac{n}{2} \rceil$.

Theorem 4.2. *For the cycle* C_n,

$$\gamma_s[M_e(C_n)] = \begin{cases} \lceil \frac{n}{2} \rceil + 1 & \textit{if n=2k, } k \geq 2. \\ \\ \lceil \frac{n}{2} \rceil & \textit{if n=2k+1, } k \geq 1. \end{cases}$$

Proof. Consider $G = C_n$, $V(C_n) = \{v_i, 1 \leq i \leq n\}$ and $e_i = v_i v_{i+1}; 1 \leq i \leq n-1$. Let D be the dominating set of $M_e(C_n)$ and is defined as follows.

$$D = \begin{cases} e_1^{'}, e_3^{'}, ...e_{n-1}^{'} & \text{if n=2k, k} \geq 2. \\ \\ e_1^{'}, e_3^{'}, ...e_{n}^{'} & \text{if n=2k+1, k} \geq 1. \end{cases}$$

Clearly, D is $\gamma_s - set$ for $n = 2k+1$ but D is not for $n = 2k$. Further, consider $D^{'} = D \cup \{e_n^{'}\}$ is a set for $n = 2k$ such that $V[M_e(C_n)] - D^{'}$ is disconnected. Thus

$$\gamma_s[M_e(C_n)] = \begin{cases} \lceil \frac{n}{2} \rceil + 1 & \text{if n=2k, k} \geq 2. \\ \\ \lceil \frac{n}{2} \rceil & \text{if n=2k+1, k} \geq 1. \end{cases}$$

Theorem 4.3. *For any graph* G, $\gamma_s[M_e(G)] \geq \gamma[M_e(G)]$.

Proof. From $M_e(G)$ definition, $V[M_e(G)] = V^{'} \cup E^{'} \cup R^{'}$. Let the dominating set of $M_e(G)$ be $D = \{u_i^{'}/u_i^{'} \in V[M_e(G)]\}$. We shall prove this in the below cases.

Case 1. Let $G = P_n$. By the Theorem 2.2 and Theorem 4.1, which implies $\gamma_s[M_e(P_n)] \geq \gamma[M_e(P_n)]$.

Case 2. Assume that G be a tree and D be the dominating set of $M_e(G)$. By Observation 3.2, $\gamma_s[M_e(T)] = \gamma[M_e(T)]$ then D itself is a $\gamma_s - set$. Hence the result follows.

Case 3. Let $G = C_n$. By the Theorem 2.3 and Theorem 4.2, we can say that $\gamma_s[M_e(C_n)] \geq \gamma[M_e(C_n)]$.

Case 4. Let G be any graph. By Theorem 2.2, Theorem 4.1, Observation 3.2, Theorem 2.3 and Theorem 4.2, it follows that $\gamma_s[M_e(G)] \geq \gamma[M_e(G)]$.

It follows that from the above cases, $\gamma_s[M_e(G)] \geq \gamma[M_e(G)]$.

Theorem 4.4. *For any graph $G(p,q)$, $\gamma_s[M_e(G)] \geq \lceil \frac{p}{1+\triangle(G)} \rceil$.*

Proof. By Theorem 2.4,

$$\gamma[M_e(G)] \geq \lceil \frac{p}{1+\triangle(G)} \rceil \tag{4.1}$$

By Theorem 4.3,

$$\gamma_s[M_e(G)] \geq \gamma[M_e(G)] \tag{4.2}$$

From equation (4.1) and (4.2),

$$\gamma_s[M_e(G)] \geq \lceil \frac{p}{1+\triangle(G)} \rceil \tag{4.3}$$

Theorem 4.5. $\gamma_s[M_e(G)] \geq \lceil \frac{diam(G)+1}{2} \rceil$ *for every graph $G(p,q)$.*

Proof. Let $V(G) = \{v_1, v_2, v_3, ...v_p\}$ such that $\exists\ u, v \in V(G)$ and $d(u,v)$ forms a diametral path in G. Clearly, $d(u,v) = diam(G)$. Consider the set D be the dominating set of $M_e(G)$. If there are at least two components in $\langle V[M_e(G)] - D\rangle$ then $\langle D\rangle$ itself is the $\gamma_s - set$ of $M_e(G)$. If not, $\exists\ \{e_j^{'}\} \in V[M_e(G)] - D$ having maximum degree such that $\langle V[M_e(G)] - D \cup \{e_j^{'}\}\rangle$ is disconnected. Clearly, $D \cup \{e_j^{'}\}$ forms a $\gamma_s - set$ of $M_e(G)$. Therefore the diametral path contains at most $\gamma_s[M_e(G)] - 1$ edges connecting the neighbourhood of the vertices of $D \cup \{e_j^{'}\}$. Hence $\gamma_s[M_e(G)] + \gamma_s[M_e(G)] - 1 \geq diam(G)$ which gives $\gamma_s[M_e(G)] \geq \lceil \frac{diam(G)+1}{2} \rceil$.

Theorem 4.6. *For any graph $G(p,q)$, $\gamma_s[M_e(G)] \geq \alpha_1(G)$.*

Proof. Suppose the minimum set of edges in G be $E_1 = \{e_1, e_2, e_3, ...e_k, 1 \leq k \leq q\}$ such that $|E_1| = \alpha_1(G)$. In $M_e(G)$, let D be the dominating set. If at least two components are found in $\langle V[M_e(G)] - D\rangle$ then $\langle D\rangle$ is the $\gamma_s - set$ of $M_e(G)$. If not, $\exists\ \{e_j^{'}\} \in V[M_e(G)] - D$ with a maximum degree such that $\langle V[M_e(G)] - D \cup \{e_j^{'}\}\rangle$ has more than one component. Evidently, $D \cup \{e_j^{'}\}$ forms a $\gamma_s - set$ of $M_e(G)$. We can say that, $\gamma_s[M_e(G)] \geq \alpha_1(G)$.

Theorem 4.7. *Let $G(p,q)$ be any graph, $\gamma_s[M_e(G)] \leq q$.*

Proof. Consider G be a any graph such that $|E| = q$. Let D be a dominating set in $M_e(G)$. If $\langle V[M_e(G)] - D\rangle$ is disconnected then $\langle D\rangle$ itself forms a $\gamma_s - set$ of $M_e(G)$. Otherwise, $\exists\ e_j^{'} \in V[M_e(G)] - D$ having maximum degree such that $\langle V[M_e(G)] - (D \cup \{e_j^{'}\})\rangle$ has more than one component. Evidently, $D \cup \{e_j^{'}\}$ forms a $\gamma_s - set$ of $M_e(G)$. Clearly, $|D| \leq |E|$. Therefore, $\gamma_s[M_e(G)] \leq q$.

5. Conclusion

In this paper we established some domination results on edge semi-middle graphs. Many bounds on domination number of edge semi-middle graph are obtained.

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