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ON CERTAIN TYPES OF NEUTROSOPHIC FUZZY GRAPHS

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ABSTRACT. In this paper, we introduce some types of NF graphs and operations. Also we define the partial NF subgraph, spanning NF subgraph, strong degree of the vertex, total strong degree of the vertex and its properties are included.

1. INTRODUCTION

The notion of neutrosophic sets (NSs) was proposed by Smarandache [48] as a generalization of the fuzzy sets [54], intuitionistic fuzzy sets [14], interval valued fuzzy set [49] and interval-valued intuitionistic fuzzy sets [15] theories. The neutrosophic set is a powerful mathematical tool for dealing with incomplete, indeterminate and inconsistent information in real world. The neutrosophic sets are characterized by a truth-membership function (t), an indeterminacy-membership function (i) and a falsity- membership function (f) independently, which are within the real standard or nonstandard unit interval $]^{-}0, 1^{+}[$. In order to conveniently use NS in real life applications, Wang et al. [50] introduced the concept of the single-valued neutrosophic set (SVNS), as a subclass of the neutrosophic sets. The same authors [51] introduced the concept of the interval valued neutrosophic set (IVNS), which is more precise and flexible than the single valued neutrosophic set. The IVNS is a generalization of the single valued neutrosophic set, in which the three membership functions are independent and their value belong to the unit interval $[0, 1]$. Graph theory has now become a major branch of applied mathematics and it is generally regarded as a branch of combinatorics. Graph is a widely used tool for solving combinatorial problems in different areas such as geometry, algebra, number theory, topology, optimization and computer science. If one has uncertainty regarding either the set of vertices or edges, or both, the model becomes a fuzzy graph. The extension of fuzzy graph [16, 29, 31] theory have been developed by several researchers, for instance vague graphs [17], considering the vertex sets and edge sets as vague sets; intuitionistic fuzzy graphs [2, 30, 34], considering the vertex sets and edge sets as intuitionistic fuzzy sets; interval valued fuzzy graphs [3, 4, 10, 11], considering the vertex sets and edge sets as interval valued fuzzy sets; interval valued intuitionistic fuzzy graphs [27], considering the vertex sets and edge sets as interval valued intuitionistic fuzzy sets; bipolar fuzzy graphs [5, 6, 8, 9], considering the vertex sets and edge sets as bipolar fuzzy sets; m-polar fuzzy graphs [7], considering the vertex sets and edge sets as m-polar fuzzy sets. But, if the relations between

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nodes (or vertices) in problems are indeterminate, the fuzzy graphs and their extensions fail. For this purpose, Smarandache [26, 44, 45, 47], defined four main categories of neutrosophic graphs; two are based on literal indeterminacy (I), called: I-edge neutrosophic graph and I-vertex neutrosophic graph, deeply studied and gaining popularity among the researchers due to their applications via real world problems [26,28], the two others are based on (t, i, f) components, called: (t, i, f)-edge neutrosophic graph and (t, i, f)-vertex neutrosophic graph, concepts not developed at all by now. Broumi et al. [19] introduced a third neutrosophic graph model, which allows the attachment of truth-membership (t), indeterminacy-membership (i) and falsity-membership degrees (f) both to vertices and edges, and investigated some of their properties. The third neutrosophic graph model is called the single valued neutrosophic graph (SVNG for short). The single valued neutrosophic graph is a generalization of fuzzy graph and intuitionistic fuzzy graph. Also, the same authors [18] introduced neighborhood degree of a vertex and closed neighborhood degree of a vertex in single valued neutrosophic graph as a generalization of neighborhood degree of a vertex and closed neighborhood degree of a vertex in fuzzy graph and intuitionistic fuzzy graph. Recently, Broumi et al. [22, 24, 25] introduced the concept of interval valued neutrosophic graph as a generalization of fuzzy graph, intuitionistic fuzzy graph and single valued neutrosophic graph and discussed some of their properties with proof and examples. Shannon and Atanassov [43] presented the concept of relationship between IFS. Then, they have introduced the concept of intuitionistic fuzzy graphs and presented many theorems in [43]. Parvathi et al. [35–37] proposed some operations between two intuitionistic fuzzy graphs. In [38] Rashmanlou et al. proposed many products operations such as lexicographic, direct product, strong product, and semi-strong product on intuitionistic fuzzy graphs. They have described the cartesian production, join, composition and union on intuitionistic fuzzy graphs in their paper. For further study on intuitionistic fuzzy graphs, please refer to [4,38,39,42]. Smarandache [44] has introduced the n-SuperHyperGraph, with super-vertices that is the most general form of graph as today. Akram et al. [5–7] have introduced the idea of pythagorean fuzzy graph. They have described the several applications of pythagorean fuzzy graph in their paper. Neutrosophic graph [21] is used to model many real-world problem which consists of inconsistent information. Recently, many scientists have researched on graph in neutrosophic environment, for instance, Yang et al. [53], Akram [12,13], Ye [53], Naz et al. [29] and Broumi [18]. Throughout the paper we denote Neutrosophic fuzzy graph by NF-graph. In this paper, we introduce some types of NF graphs and operations. Also we define the partial NF subgraph, spanning NF subgraph, strong degree of the vertex, total strong degree of the vertex, neutrosophic sum and its properties are included.

2. Preliminaries

In this section, we mainly recall some notions related to neutrosophic sets, single valued neutrosophic sets, single valued neutrosophic graphs, relevant to the present article. See [48, 50] for further details and background.

Definition 2.1. [48] Let X be a space of points (objects) with generic elements in X denoted by x; then, the neutrosophic set A (in short NS A) is an object having the form $A = \{[x : T_A(x), I_A(x), F_A(x)], x \in X\}$ where the functions $T_A : V \to [0, 1]$, $I_A : V \to [0, 1]$, $F_A : V \to [0, 1]$ define respectively a truth membership function, an indeterminacy membership function, a falsity membership function of the element $x \in X$ to the set A with the condition:
$$^{-}0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+}.$$
The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]^{-}0, 1^{+}[$.

Since it is difficult to apply NSs to practical problems, Wang et al. [50] introduced the concept of SVNS, which is an instance of a NS, and can be used in real scientific and engineering applications.

Definition 2.2. [50] Let X be a non-empty set. The neutrosophic set A (NS set) is written as $A = \{[x : T_A(x), I_A(x), F_A(x)], x \in X\}$ and the functions $T_A : V \to [0,1]$, $I_A : V \to [0,1]$, $F_A : V \to [0,1]$ denotes the degree of truth membership , the degree of indeterminacy membership , the degree of falsity membership of elements of X respectively and for all $x \in X$ we have

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$$

Definition 2.3. [19] A single valued neutrosophic graph (SVN-graph) with underlying set V is defined to be a pair $G = (A, B)$, where :

(1) The functions $T_A : V \to [0,1]$, $I_A : V \to [0,1]$ and $F_A : V \to [0,1]$ denote the degree of truth-membership, degree of indeterminacy-membership and falsity-membership of the element $v_i \in V$ respectively, and $0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3$, for all $v_i \in V$.

(2) The functions $T_B : E \subseteq V \times V \to [0,1]$, $I_B : E \subseteq V \times V \to [0,1]$ and $F_B : E \subseteq V \times V \to [0,1]$ are defined by

$$T_B(v_i, v_j) \leq min[T_A(v_i), T_A(v_j)],$$

$$I_B(v_i, v_j) \geq max[I_A(v_i I_A(v_j)],$$

$$F_B(v_i, v_j) \geq max[F_A(v_i), F_A(v_j)],$$

denoting the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i, v_j) \in E$ respectively, where :

$$0 \leq T_B(v_i, v_j) + I_B(v_i, v_j) + F_B(v_i, v_j) \leq 3,$$

for all $(v_i, v_j) \in E$ ($i, j = 1, 2, \ldots, n$).

We call A the single valued neutrosophic vertex set of V, and B the single valued neutrosophic edge set of E.

Definition 2.4. [26] Let $G = (V, E)$ be a graph where V, E are the set of its vertices and edges respectively. The fuzzy graph is a pair of functions $F_G = (\sigma, \mu)$ where σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ . i.e $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ such that $\mu(u,v) \leq \mu(u) \wedge \mu(v)$ for all $u, v \in V$ where (u,v) denotes the edge between u and v and $\mu(u) \wedge \mu(v)$ denotes the minimum of $\mu(u)$ and $\mu(v)$. σ is called the fuzzy vertex set of V and μ is called the fuzzy edge set of E.

Definition 2.5. [47] If X is any crisp set, and A, B are two neutrosophic sets. we say that $A =_N B$ if $T_A(x) = T_B(x), I_A(x) = I_B(x), F_A(x) = F_B(x)$ for all $x \in X$ and $A \leq_N B$ if $T_A(x) \leq T_B(x), I_A(x) \leq I_B(x), F_A(x) \leq F_B(x)$ for all $x \in X$

3. Some types of NF-graphs and operations

Definition 3.1. Consider the graph $G = (V, E)$ where V and E is the set of its vertex and edges respectively. The neutrosophic fuzzy graph (NF-graph) $NF_G = (A, B)$ where $A = [T_A, I_A, F_A]$ and the functions $T_A : V \to [0,1]$, $I_A : V \to [0,1]$, $F_A : V \to [0,1]$ denotes the degree of truth membership vertex, the degree of indeterminacy membership vertex, the degree of falsity membership vertex respectively and for all $v \in V$ we have

$$0 \leq T_A(v) + I_A(v) + F_A(v) \leq 3$$

and $B = [T_B, I_B, F_B]$ where the functions $T_B : V \times V \to [0,1]$, $I_B : V \times V \to [0,1]$ and $F_B : V \times V \to [0,1]$ defined by

$$T_B(v, w) \leq T_A(v) \times T_A(w),$$

$$I_B(v, w) \leq I_A(v) \times I_A(w),$$

$$F_B(v,w) \le F_A(v) \times F_A(w),$$

and

$$0 \le T_B(v,w) + I_B(v,w) + F_B(v,w) \le 3,$$

for all $v,w \in V$. $A = [T_A, I_A, F_A]$ and $B = [T_B, I_B, F_B]$ are called the vertex set and the edge set of $NF_G = (A,B)$ respectively.

Definition 3.2. A NF-graph $NF_G = (A,B)$ is called complete if

$$T_B(v,w) = T_A(v) \times T_A(w),$$
$$I_B(v,w) = I_A(v) \times I_A(w),$$
$$F_B(v,w) = F_A(v) \times F_A(w)$$

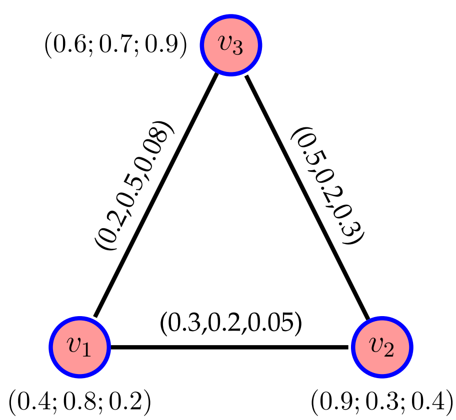
for all $v,w \in V$.

Definition 3.3. The complement of a NF-graph $NF_G = (A,B)$ is the NF-graph $\overline{NF_G} = (\overline{A}, \overline{B})$ if
$V = \overline{V}$, where V, $\overline{V}$ are the vertex set of G and its complement $\overline{G}$ respectively,
$T_A(v) = T_{\overline{A}}(v)$, $I_A(v) = I_{\overline{A}}(v)$, $F_A(v) = F_{\overline{A}}(v)$.

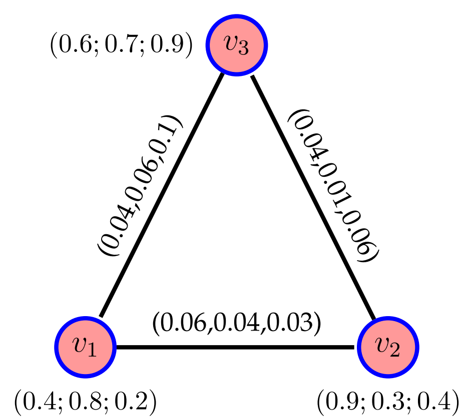
$$T_{\overline{B}}(v,w) = T_A(v) \times T_A(w) - T_B(v,w),$$
$$I_{\overline{B}}(v,w) = I_A(v) \times I_A(w) - I_B(v,w),$$
$$F_{\overline{B}}(v,w) = F_A(v) \times F_A(w) - F_B(v,w),$$

for all $v,w \in V$.

Example 3.4. Consider the following graph $G = (V,E)$ where $V = \{v_1, v_2, v_3\}$ and
$E = \{(v_1,v_2), (v_1,v_3), (v_3,v_2)\}$. The NF-graph $NF_G = (A,B)$ is defined as in the following diagram



(A) The NF-graph $NF_G = (A,B)$



(B) The NF-graph $\overline{NF_G} = (\overline{A}, \overline{B})$

FIGURE 1. Two NF-graphs one is the complement of the other.

Remark 3.5. It is obvious that $\overline{\overline{NF_G}} = NF_G$.

Definition 3.6. Let $G_1 = (V_1, E_1)$, $G_2 = (V_2, E_2)$ and $G_2 = (V, E)$ be any crisp graphs. The neutrosophic sum of the NF-graph $NF_{G_1} = (A_1, B_1)$ and $NF_{G_2} = (A_2, B_2)$ is the NF-graph $NF_G = (A,B)$ (denoted $NF_G = NF_{G_1} \oplus NF_{G_2}$ if the following conditions are true:

(1) $V_1 = V_2 = V$, where $T_{A_1}(v) = T_{A_2}(v)$, $I_{A_1}(v) = I_{A_2}(v)$, $F_{A_1}(v) = F_{A_2}(v)$ for all $v \in V$,

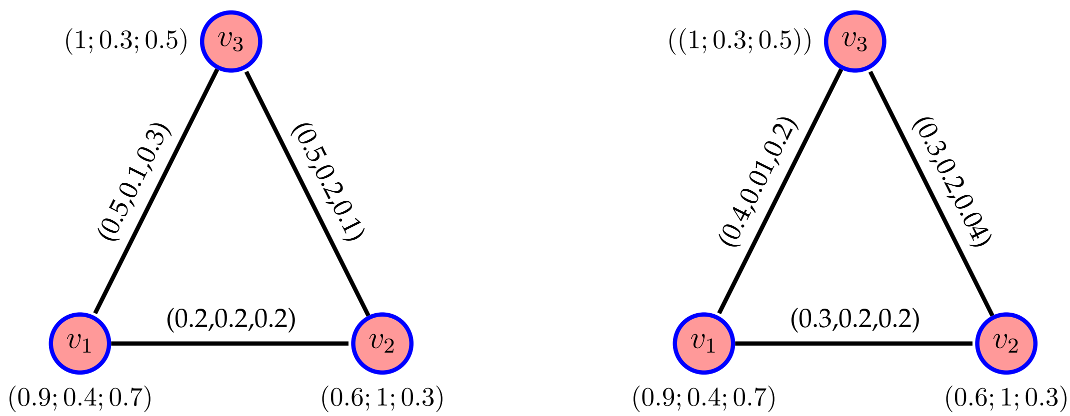
(2) $E = E_1 \cup E_2$ and for all $(v, w) \in E$.

$$T_B(v,w) = \begin{cases} T_{B_1}(v,w) + T_{B_2}(v,w) - T_A(v) \times T_A(w) & : \quad T_{B_1}(v,w) + T_{B_2}(v,w) > T_A(v) \times T_A(w); \\ T_{B_1}(v,w) + T_{B_2}(v,w) & : \quad otherwise, \end{cases}$$

$$I_B(v,w) = \begin{cases} I_{B_1}(v,w) + I_{B_2}(v,w) - I_A(v) \times I_A(w) & : \quad I_{B_1}(v,w) + I_{B_2}(v,w) > I_A(v) \times I_A(w); \\ I_{B_1}(v,w) + I_{B_2}(v,w) & : \quad otherwise, \end{cases}$$

$$F_B(v,w) = \begin{cases} F_{B_1}(v,w) + F_{B_2}(v,w) - F_A(v) \times F_A(w) & : \quad F_{B_1}(v,w) + F_{B_2}(v,w) > F_A(v) \times F_A(w); \\ F_{B_1}(v,w) + F_{B_2}(v,w) & : \quad otherwise, \end{cases}$$

Example 3.7. Consider the following NF-graphs



(A) The NF-graph $NF_{G_1} = (A_1, B_1)$ (B) The NF-graph $NF_{G_2} = (A_2, B_2)$

FIGURE 2. Two NF-graphs.

Then the neotrosophic sum of NF_{G_1} and NF_{G_2} is the NF-graph $NF_G = (A, B)$ as follows:

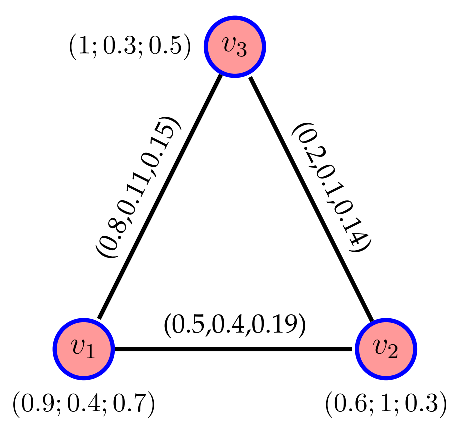


FIGURE 3. The NF-graph $NF_G = NF_{G_1} \oplus NF_{G_2}$

Proposition 3.8. *Let $G_1 = (V, E_1)$, $G_2 = (V, E_2)$ be two crisp graphs with the corresponding NF-graphs $NF_{G_1} = (A_1, B_1)$, $NF_{G_2} = (A_2, B_2)$. Then their neutrosophic sum is complete if and only if one of them is neutrosophic complement of the other.*

Proof. Let $NF_{G_1} = (A_1, B_1)$ and $NF_{G_2} = (A_2, B_2)$ be neutrosophic complement of each other and suppose that $NF_H = (A, B)$ is their neutrosophic sum that is $NF_H = NF_{G_1} \oplus NF_{G_2}$. Hence, by Definition 3.2, we have

$$T_{B_1}(v, w) = T_A(v) \times T_A(w) - T_{B_2}(v, w),$$
$$I_{B_1}(v, w) = I_A(v) \times I_A(w) - I_{B_2}(v, w),$$
$$F_{B_1}(v, w) = F_A(v) \times F_A(w) - F_{B_2}(v, w),$$

for all $(v, w) \in E$.

From Definition 3.6, we have

$$T_B(v, w) = T_{B_1}(v, w) + T_{B_2}(v, w),$$
$$I_B(v, w) = I_{B_1}(v, w) + I_{B_2}(v, w),$$
$$F_B(v, w) = F_{B_1}(v, w) + F_{B_2}(v, w),$$

for all $(v, w) \in E$. Substituting we obtain

$$T_B(v, w) = T_A(v) \times T_A(w),$$
$$I_B(v, w) = I_A(v) \times I_A(w),$$
$$F_B(v, w) = F_A(v) \times F_A(w),$$

for all $(v, w) \in E$. This implies that $NF_H = (A, B)$ is complete.
Conversely, by reversing the steps, we get the result. □

Proposition 3.9. *Let $\overline{NF_G} = (\overline{A}, \overline{B})$ be the complement of the NF-graph $NF_G = (A, B)$. Then $NF_G = (A, B)$ is a complete NF-graph if and only if*

$$T_{\overline{B}}(v, w) = I_{\overline{B}}(v, w) = F_{\overline{B}}(v, w) = 0 \quad \forall v, w \in V.$$

Proof. Suppose that $NF_G = (A, B)$ is a complete NF-graph, then by Definition 3.2, we have

$$T_B(v, w) = T_A(v) \times T_A(w),$$
$$I_B(v, w) = I_A(v) \times I_A(w),$$
$$F_B(v, w) = F_A(v) \times F_A(w)$$

for all $v, w \in V$. From Definition 3.3,

$$T_{\overline{B}}(v, w) = T_A(v) \times T_A(w) - T_B(v, w),$$
$$I_{\overline{B}}(v, w) = I_A(v) \times I_A(w) - I_B(v, w),$$
$$F_{\overline{B}}(v, w) = F_A(v) \times F_A(w) - F_B(v, w),$$

for all $v, w \in V$. Hence,

$$T_{\overline{B}}(v, w) = I_{\overline{B}}(v, w) = F_{\overline{B}}(v, w) = 0 \quad \forall v, w \in V.$$

conversely, if

$$T_{\overline{B}}(v, w) = I_{\overline{B}}(v, w) = F_{\overline{B}}(v, w) = 0 \quad \forall v, w \in V.$$

Then we obtain that

$$0 = T_A(v) \times T_A(w) - T_B(v, w),$$
$$0 = I_A(v) \times I_A(w) - I_B(v, w),$$
$$0 = F_A(v) \times F_A(w) - F_B(v, w),$$

for all $v, w \in V$. Hence,

$$T_B(v, w) = T_A(v) \times T_A(w),$$

$$I_B(v,w) = I_A(v) \times I_A(w),$$

$$F_B(v,w) = F_A(v) \times F_A(w)$$

for all $v, w \in V$. Implies $NF_G = (A, B)$ is a complete NF-graph. □

Definition 3.10. Let $H = (V_1, E_1)$ and $G = (V_2, E_2)$ be any two graphs. A NF-graph $NF_H = (A_1, B_1)$ is partial NF-subgraph of the NF-graph $NF_G = (A_2, B_2)$ if the following conditions are true:

(1) $V_1 \subseteq V_2$, where $T_{A_1}(v) \leq T_{A_2}(v)$, $I_{A_1}(v) \leq I_{A_2}(v)$, $F_{A_1}(v) \leq F_{A_2}(v)$ for all $v \in V_1$,
(2) $E_1 \subseteq E_2$, where $T_{B_1}(v,w) \leq T_{B_2}(v,w)$, $I_{B_1}(v,w) \leq I_{B_2}(v,w)$, $F_{B_1}(v,w) \leq F_{B_2}(v,w)$ for all $(v,w) \in E_1$.

Definition 3.11. Let $H = (V_1, E_1)$ and $G = (V_2, E_2)$ be any two graphs. A NF-graph $NF_H = (A_1, B_1)$ is called a NF-subgraph of the NF-graph $NF_G = (A_2, B_2)$ if the following conditions are true:

(1) $V_1 \subseteq V_2$, where $T_{A_1}(v) = T_{A_2}(v)$, $I_{A_1}(v) = I_{A_2}(v)$, $F_{A_1}(v) = F_{A_2}(v)$ for all $v \in V_1$,
(2) $E_1 \subseteq E_2$, where $T_{B_1}(v,w) = T_{B_2}(v,w)$, $I_{B_1}(v,w) = I_{B_2}(v,w)$, $F_{B_1}(v,w) = F_{B_2}(v,w)$ for all $(v,w) \in E_1$.

Definition 3.12. Let $H = (V, E_1)$ and $G = (V, E_2)$ be any two graphs. A NF-graph $NF_H = (A_1, B_1)$ is called a spanning NF-subgraph of the NF-graph $NF_G = (A_2, B_2)$ if the following conditions are true:

(1) $T_{A_1}(v) = T_{A_2}(v)$, $I_{A_1}(v) = I_{A_2}(v)$, $F_{A_1}(v) = F_{A_2}(v)$ for all $v \in V$,
(2) $E_1 \subseteq E_2$, where $T_{B_1}(v,w) = T_{B_2}(v,w)$, $I_{B_1}(v,w) = I_{B_2}(v,w)$, $F_{B_1}(v,w) = F_{B_2}(v,w)$ for all $(v,w) \in E_1$.

Definition 3.13. Let $H_1 = (V, E_1)$ and $H_2 = (V, E_2)$ be two crisp graphs and $NF_{H_1} = (A_1, B_1)$, $NF_{H_2} = (A_2, B_2)$ are two spanning NF-subgraphs of the NF-graph $NF_G = (A, B)$, we call that $NF_{H_1} \sqcap NF_{H_2} = \Phi$ if $E_1 \cap E_2 = \phi$.
Moreover, if $G = (V, E)$, $H_1(V, E_1)$, $H_2(V, E_2)$, $\ldots$ are crisp graphs such that $NF_{H_1} = (A_1, B_1)$, $NF_{H_2} = (A_2, B_2)$, $\ldots$ are spanning NF-subgraphs of the NF-graph $NF_G = (A, B)$. If $E = \bigcup_{i=1}^{\infty} E_i$, then we say that

$$NF_G = \bigsqcup_{i=1}^{\infty} NF_{H_i}$$

.

Proposition 3.14. *Let $G = (V, E)$ be any crisp graph. Then the NF-graph $NF_G = (A, B)$ is a complete NF-graph if and only if $NF_H \oplus NF_H = NF_H$ for every spanning NF-subgraph NF_H of the NF-graph NF_G.*

Proof. Suppose that $NF_G = (A, B)$ is complete and NF_H is any spanning subgraph of it. Since $NF_G = (A, B)$ is complete, so by Definition 3.2, we have

$$T_B(v,w) = T_A(v) \times T_A(w),$$

$$I_B(v,w) = I_A(v) \times I_A(w),$$

$$F_B(v,w) = F_A(v) \times F_A(w)$$

for all $v, w \in V$. Since NF_H is a spanning subgraph of NF_G, so we obtain that

$$T_{B_1}(v,w) = T_A(v) \times T_A(w),$$

$$I_{B_1}(v,w) = I_A(v) \times I_A(w),$$

$$F_{B_1}(v,w) = F_A(v) \times F_A(w)$$

for all $v, w \in V$. Hence, by Definition 3.6, we deduce that the truth, intermidacy and fulsity membership of $NF_H \oplus NF_H$ is

$$T_{B_1}(v,w) = 2T_A(v) \times T_A(w) - T_A(v) \times T_A(w) = T_A(v) \times T_A(w),$$

$$I_{B_1}(v,w) = 2I_A(v) \times I_A(w) - I_A(v) \times I_A(w) = I_A(v) \times I_A(w),$$

$$F_{B_1}(v,w) = 2F_A(v) \times F_A(w) - F_A(v) \times F_A(w) = F_A(v) \times F_A(w)$$

for all $v, w \in V$. Hence, $NF_H \oplus NF_H = NF_H$.
Conversely, Since $NF_H \oplus NF_H = NF_H$ for every spanning NF-subgraph NF_H of the NF-graph NF_G and NF_G is a spanning NF-subgraph of itself, so by hypothesis $NF_G \oplus NF_G = NF_G$. Therefore, by Definition 3.6, we get

$$T_B(v,w) + T_B(v,w) - T_A(v) \times T_A(w) = T_B(v,w),$$

$$I_B(v,w) + I_B(v,w) - I_A(v) \times I_A(w) = I_B(v,w),$$

$$F_B(v,w) + F_B(v,w) - F_A(v) \times F_A(w) = F_B(v,w)$$

for all $v, w \in V$. This implies that

$$T_B(v,w) = T_A(v) \times T_A(w),$$

$$I_B(v,w) = I_A(v) \times I_A(w),$$

$$F_B(v,w) = F_A(v) \times F_A(w)$$

for all $v, w \in V$. Therefore, NF_G is complete. □

4. Degrees and strong degrees in NF-graphs

Definition 4.1. Consider the graph $G = (V, E)$ where V and E is the set of its vertex and edges respectively. $NF_G = (A, B)$ be the NF-graph. Then the degree of the vertex $v \in V$ is denoted by $d(v)$ and defined as $d(v) = [d_T(v), d_I(v), d_F(v)]$ (or $d_G(v) = [d_{T_G}(v), d_{I_G}(v), d_{F_G}(v)]$) where

$$d_T(v) = \sum_{v \neq w} T_B(v,w), \qquad d_I(v) = \sum_{v \neq w} I_B(v,w), \qquad d_F(v) = \sum_{v \neq w} F_B(v,w)$$

denotes the degree of truth membership vertex, the degree of indeterminacy membership vertex, the degree of falsity membership vertex respectively. If $T_B(v,w) = I_B(v,w) = F_B(v,w) = 0$ for $v, w \in V$, then we say that they are not adjacent.

Definition 4.2. Consider the graph $G = (V, E)$ where V and E is the set of its vertex and edges respectively. $NF_G = (A, B)$ be the NF-graph. Then the total degree of the vertex $v \in V$ is denoted by $td(v)$ and defined as $td(v) = [td_T(v), td_I(v), td_F(v)]$ where

$$td_T(v) = \sum_{v \neq w} T_B(v,w) + T_A(v), \qquad td_I(v) = \sum_{v \neq w} I_B(v,w) + I_A(v), \qquad td_F(v) = \sum_{v \neq w} F_B(v,w) + F_A(v).$$

Definition 4.3. Consider the graph $G = (V, E)$ where V and E are the set of its vertex and edges respectively. The neighborhood of a vertex $v \in V$ is denoted by $\mathcal{N}(v)$ and is defined as $\mathcal{N}(v) = \{w \in V : (v, w) \in E\}$.

Example 4.4. Consider the following graph $G = (V, E)$ where $V = \{v_1, v_2, v_3, v_4\}$ and
$E = \{(v_1, v_2), (v_1, v_3), (v_2, v_4), (v_3, v_4), (v_3, v_2)\}$. The NF-graph $NF_G = (A, B)$ is defined as in the following diagram

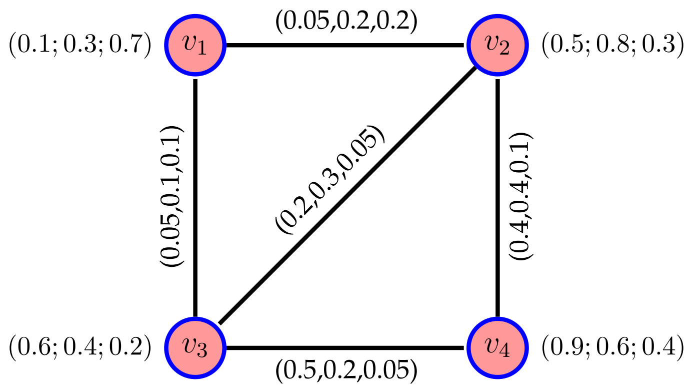


FIGURE 4. NF-graph $NF_G = (A, B)$

By simple calculation, we get
$d(v_1) = [d_T(v_1), d_I(v_1), d_F(v_1)] = [0.1, 0.3, 0.3]$, $d(v_2) = [d_T(v_2), d_I(v_2), d_F(v_2)] = [0.65, 0.9, 0.35]$
$d(v_3) = [d_T(v_3), d_I(v_3), d_F(v_3)] = [0.75, 0.6, 0.2]$, $d(v_4) = [d_T(v_4), d_I(v_4), d_F(v_4)] = [0.9, 0.6, 0.15]$.
$td(v_1) = [0.2, 0.6, 1]$, $td(v_2) = [1.15, 1.7, 0.65]$
$td(v_3) = [1.35, 1, 0.4]$, $td(v_4) = [1.8, 1.2, 0.55]$.

Proposition 4.5. *Let $G = (V, E)$ where V is its vertex set and E its edge set. $NF_G = (A, B)$ be the NF-graph. Then the degree of truth membership vertex, the degree of indeterminacy membership vertex, the degree of falsity membership vertex respectively is of the vertex $v \in V$ is given by*

$$d_T(v) \leq T_A(v)(\sum_{w\in\mathcal{N}(v)} T_A(w)),$$

$$d_I(v) \leq I_A(v)(\sum_{w\in\mathcal{N}(v)} I_A(w)),$$

$$d_F(v) \leq F_A(v)(\sum_{w\in\mathcal{N}(v)} F_A(w))$$

and

$$td_T(v) \leq T_A(v)(\sum_{w\in\mathcal{N}(v)} T_A(w) + 1),$$

$$td_I(v) \leq I_A(v)(\sum_{w\in\mathcal{N}(v)} I_A(w) + 1),$$

$$td_F(v) \leq F_A(v)(\sum_{w\in\mathcal{N}(v)} F_A(w) + 1)$$

Proof. From Definition 4.1, we have

$$d_T(v) = \sum_{v\neq w} T_B(v, w), \qquad d_I(v) = \sum_{v\neq w} I_B(v, w), \qquad d_F(v) = \sum_{v\neq w} F_B(v, w)$$

and from Definition 3.1, we have

$$T_B(v, w) \leq T_A(v) \times T_A(w),$$

$$I_B(v, w) \leq I_A(v) \times I_A(w),$$

$$F_B(v, w) \leq F_A(v) \times F_A(w).$$

Since $T_B(v,w) = I_B(v,w) = F_B(v,w) = 0 \quad \forall (v,w) \notin E$ Therefore,

$$d_T(v) \leq \sum_{w\in\mathcal{N}(v)} T_A(v)(T_A(w)) = T_A(v)(\sum_{w\in\mathcal{N}(v)} T_A(w)),$$

$$d_I(v) \leq \sum_{w\in\mathcal{N}(v)} I_A(v)(I_A(w)) = I_A(v)(\sum_{w\in\mathcal{N}(v)} I_A(w)),$$

$$d_F(v) \leq \sum_{w\in\mathcal{N}(v)} F_A(v)(F_A(w)) = F_A(v)(\sum_{w\in\mathcal{N}(v)} F_A(w)).$$

From Definition 4.2, and above equations, we obtain that

$$td_T(v) \leq \sum_{w\in\mathcal{N}(v)} T_A(v)(T_A(w)) + T_A(v) = T_A(v)(\sum_{w\in\mathcal{N}(v)} T_A(w) + 1),$$

$$td_I(v) \leq \sum_{w\in\mathcal{N}(v)} I_A(v)(I_A(w)) + I_A(v) = I_A(v)(\sum_{w\in\mathcal{N}(v)} I_A(w) + 1),$$

$$td_F(v) \leq \sum_{w\in\mathcal{N}(v)} F_A(v)(F_A(w)) + F_A(v) = F_A(v)(\sum_{w\in\mathcal{N}(v)} F_A(w) + 1).$$

□

Proposition 4.6. *Let $NF_G = (A,B)$ be a complete NF-graph. Then*

$$d_T(v) = T_A(v)(\sum_{w\in\mathcal{N}(v)} T_A(w)),$$

$$d_I(v) = I_A(v)(\sum_{w\in\mathcal{N}(v)} I_A(w)),$$

$$d_F(v) = F_A(v)(\sum_{w\in\mathcal{N}(v)} F_A(w))$$

and

$$td_T(v) = T_A(v)(\sum_{w\in\mathcal{N}(v)} T_A(w) + 1),$$

$$td_I(v) = I_A(v)(\sum_{w\in\mathcal{N}(v)} I_A(w) + 1),$$

$$td_F(v) = F_A(v)(\sum_{w\in\mathcal{N}(v)} F_A(w) + 1)$$

Proof. Follows from Definition 3.2 and Proposition 4.5. □

Proposition 4.7. (1) *Let $NF_H = (C,D)$ be any partial subgraph of the NF-graph $NF_G = (A,B)$. Then*

$$d_{T_H}(v) \leq d_{T_G}(v), \quad d_{I_H}(v) \leq d_{I_G}(v), \quad d_{F_H}(v) \leq d_{F_G}(v),$$

and

$$td_{T_H}(v) \leq td_{T_G}(v), \quad td_{I_H}(v) \leq td_{I_G}(v), \quad td_{F_H}(v) \leq td_{F_G}(v),$$

for all $v \in V(H)$.

(2) *Let $G = (V,E)$ be any graph and let $\{v_1, v_2, \dots\}$ be the set of isolated vertices of G. If $H = (V_1, E_1)$ is a subgraph of G such that $V_1 = V \setminus \{v_i : i \in N\}$. Then*

$$d_{T_H}(v) = d_{T_G}(v), \quad d_{I_H}(v) = d_{I_G}(v), \quad d_{F_H}(v) = d_{F_G}(v),$$

and

$$td_{T_H}(v) = td_{T_G}(v), \quad td_{I_H}(v) = td_{I_G}(v), \quad td_{F_H}(v) = td_{F_G}(v),$$

for all $v \in V(H)$.

Proof. (1) Follows from the definition.
(2) Since $\{v_1, v_2, \dots\}$ is the set of isolated vertices of G. Hence $T_B(v, v_i) = I_B(v, v_i) = F_B(v, v_i) = 0 \quad \forall i = 1, 2, \dots$. Therefore

$$d_{T_H}(v) = d_{T_G}(v), \quad d_{I_H}(v) = d_{I_G}(v), \quad d_{F_H}(v) = d_{F_G}(v),$$

and

$$td_{T_H}(v) = td_{T_G}(v), \quad td_{I_H}(v) = td_{I_G}(v), \quad td_{F_H}(v) = td_{F_G}(v),$$

for all $v \in V(H)$. □

Proposition 4.8. *Let $G = (V, E)$ where V is its vertex set and E its edge set. $NF_G = (A, B)$ be the NF-graph. Then*

$$\sum_{v \in V} d_T(v) = 2 \sum_{v \neq w} T_B(v, w),$$

$$\sum_{v \in V} d_I(v) = 2 \sum_{v \neq w} I_B(v, w),$$

$$\sum_{v \in V} d_F(v) = 2 \sum_{v \neq w} F_B(v, w).$$

Proof. Since $T_B(v, w) = T_B(w, v)$, $I_B(v, w) = I_B(w, v)$, $F_B(v, w) = F_B(w, v)$ for every $v, w \in V$, so in $\sum_{v \in V} d_T(v)$, $\sum_{v \in V} d_I(v)$ and $\sum_{v \in V} d_F(v)$, we get $T_B(v, w) + T_B(w, v)$, $I_B(v, w) + I_B(w, v)$, $F_B(v, w) + F_B(w, v)$. Hence, the result. □

Definition 4.9. Let $G = (V, E)$ be any graph, an edge $e = (v, w) \in E$, then the corresponding NF-edge of the NF-graph $NF_G = (A, B)$ is called a strong edge (denoted by E_s), if

$$T_B(v, w) = T_A(v) \times T_A(w),$$

$$I_B(v, w) = I_A(v) \times I_A(w),$$

$$F_B(v, w) = F_A(v) \times F_A(w).$$

In this case, the vertex v is called a strong neighbor of w and conversely.
$\mathcal{SN}(v) = \{w \in V : (u, w) \in E_s\}$ is called the strong neighborhood of v. and the set $\mathcal{SN}[v] = \{v\} \cup \mathcal{SN}(v)$ is called the closed strong neighborhood of v.

Example 4.10. Consider the following graph $G = (V, E)$ where $V = \{v_1, v_2, v_3, v_4, v_5\}$ and $E = \{(v_1, v_2), (v_1, v_3), (v_2, v_4), (v_3, v_4), (v_3, v_2)\}$. The NF-graph $NF_G = (A, B)$ is defined as in the following diagram:

It is easy to see that $\mathcal{SN}(v_1) = \{v_2, v_4\}$, $\mathcal{SN}(v_2) = \{v_1, v_4, v_5\}$, $\mathcal{SN}(v_3) = \phi$, $\mathcal{SN}(v_4) = \{v_1, v_2, v_5\}$ and $\mathcal{SN}(v_5) = \{v_2, v_4\}$.

Definition 4.11. Consider the graph $G = (V, E)$ where V and E is the set of its vertex and edges respectively. $NF_G = (A, B)$ be the NF-graph. Then the strong degree of the vertex $v \in V$ is denoted by $sd(v)$ and defined as $sd(v) = [sd_T(v), sd_I(v), sd_F(v)]$ where

$$sd_T(v) = \sum_{w \in \mathcal{SN}(v)} T_B(v, w), \qquad sd_I(v) = \sum_{w \in \mathcal{SN}(v)} I_B(v, w), \qquad sd_F(v) = \sum_{w \in \mathcal{SN}(v)} F_B(v, w)$$

denotes the strong degree of truth, indeterminacy and falsity membership vertex v respectively. If $sT_B(v, w) = sI_B(v, w) = sF_B(v, w) = 0$ for some $v, w \in V$, then we say that $w \in \mathcal{SN}(v)$.

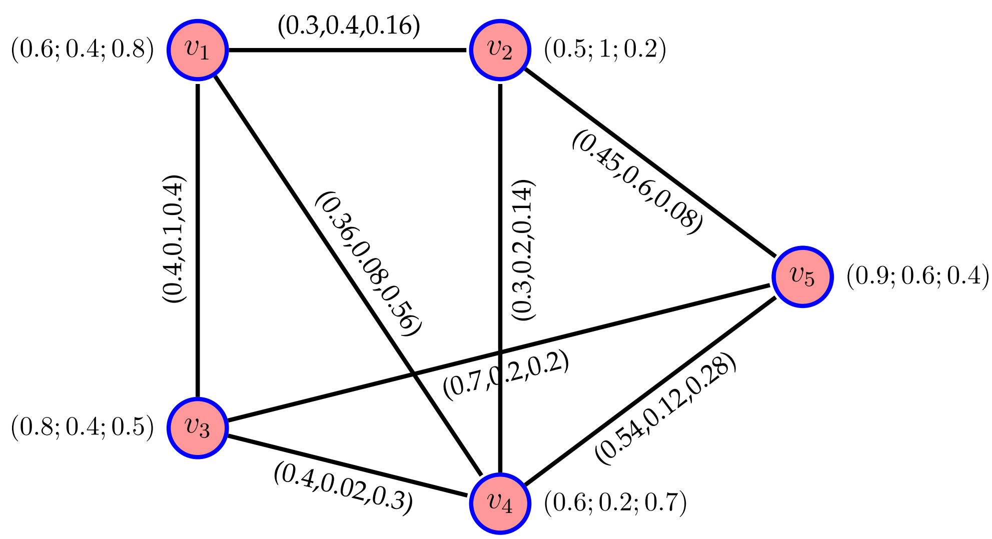


FIGURE 5. NF-graph $NF_G = (A, B)$

Definition 4.12. Consider the graph $G = (V, E)$ where V and E is the set of its vertex and edges respectively. $NF_G = (A, B)$ be the NF-graph. Then the total strong degree of the vertex $v \in V$ is denoted by $tsd(v)$ and defined as $tsd(v) = [tsd_T(v), tsd_I(v), tsd_F(v)]$ where

$$tsd_T(v) = \sum_{w \in \mathcal{SN}(v)} T_B(v, w) + T_A(v),$$

$$tsd_I(v) = \sum_{w \in \mathcal{SN}(v)} I_B(v, w) + I_A(v),$$

$$tsd_F(v) = \sum_{w \in \mathcal{SN}(v)} F_B(v, w) + F_A(v).$$

From Example 4.10, we conclude that
$sd(v_1) = [0.66, 0.48, 0.72]$, $sd(v_2) = [1.05, 1.2, 0.38]$, $sd(v_3) = [0, 0, 0]$, $sd(v_4) = [1.2, 0.3, 0.98]$ and $sd(v_5) = [0.99, 0.72, 0.36]$.
$tsd(v_1) = [1.26, 0.88, 1.52]$, $tsd(v_2) = [1.55, 2.2, 0.58]$, $tsd(v_3) = [0.8, 0.4, 0.5]$, $tsd(v_4) = [1.8, 0.5, 1.68]$ and $sd(v_5) = [1.89, 1.32, 0.76]$.

Proposition 4.13. *Let $G = (V, E)$ be any graph and $NF_G(A, B)$ is its corresponding NF-graph, then the following are true:*

(1) *If $v \in V$ is an isolated vertex, then $d(v) = [0, 0, 0]$ and $td(v) = [T_A(v), I_A(v), F_A(v)]$.*
(2) *If $\mathcal{SN}(v) = \phi$, then $sd(v) = [0, 0, 0]$ and $tsd(v) = [T_A(v), I_A(v), F_A(v)]$.*
(3) *If $v \in V$ is an isolated vertex, then $d(v) = sd(v)$ and $td(v) = tsd(v)$.*
(4) *$sd(v) \leq d(v)$ and $tsd(v) \leq td(v)$ for all $v \in V$.*
(5) *The NF-graph $NF_G(A, B)$ is complete if and only if $sd(v) = d(v)$ and $tsd(v) = td(v)$ for all $v \in V$.*

Proof.
(1), (2) The proof of (1) and (2) follow from their definitions.
(3) Follows from (1) and (2).
(4) Since $sd(v) = [sd_T(v), sd_I(v), sd_F(v)]$ and $sd_T(v), sd_I(v), sd_F(v)$ is a sum taken from only strong edges which are adjacent to v, so this sum is less than or equal to the sum taken over all edges adjacent to v.

Therefore, $sd(v) \leq d(v)$ and if we add $T_A(v), I_A(v), F_A(v)$ to both sides of the inequality respectively, then we get $tsd(v) \leq td(v)$ for all $v \in V$.

(5) Suppose that $NF_G(A,B)$ is a complete graph, then by definition every edge is strong and hence $sd(v) = d(v)$ and $tsd(v) = td(v)$ for all $v \in V$. Now if $sd(v) = d(v)$ for all $v \in V$ implies that

$$\sum_{w \in \mathcal{N}(v)} T_B(v,w) = d_T(v) = sd_T(v) = \sum_{w \in \mathcal{SN}(v)} T_B(v,w),$$

and this will be true only when $\mathcal{SN}(v) = \mathcal{N}(v)$ for all $v \in V$. Hence, every edge is strong means that

$$T_B(v,w) = T_A(v) \times T_A(w),$$

$$I_B(v,w) = I_A(v) \times I_A(w),$$

$$F_B(v,w) = F_A(v) \times F_A(w)$$

for all $(v,w) \in E$. Therefore, $NF_G(A,B)$ is a complete graph. □

Proposition 4.14. *if $G = (V,E)$, $H_1(V,E_1)$, $H_2(V,E_2)$, $\ldots$ are crisp graphs such that $NF_{H_1} = (A_1,B_1)$, $NF_{H_2} = (A_2,B_2)$, $\ldots$ are spanning NF-subgraphs of the NF-graph $NF_G = (A,B)$ with $E = \bigcup_{i=1}^{\infty} E_i$. Then*

$$NF_G = \bigsqcup_{i=1}^{\infty} NF_{H_i},$$

and

$$d_T(v) = \sum_{i=1}^{\infty} d_T^i(v) - \sum_{(u,v) \in E_i \cap E_j} T_B(u,v),$$

$$d_I(v) = \sum_{i=1}^{\infty} d_I^i(v) - \sum_{(u,v) \in E_i \cap E_j} I_B(u,v),$$

$$d_F(v) = \sum_{i=1}^{\infty} d_F^i(v) - \sum_{(u,v) \in E_i \cap E_j} F_B(u,v),$$

where $d_T^i(v)$, $d_I^i(v)$ and $d_F^i(v)$ denotes the degree of truth membership vertex, the degree of indeterminacy membership vertex, the degree of falsity membership vertex respectively in the neuotrosophic soft graph NF_{H_i}. Moreover, if E_i is pairwise disjoint for all $i = 1, 2, \ldots$, then

$$d_T(v) = \sum_{i=1}^{\infty} d_T^i(v),$$

$$d_I(v) = \sum_{i=1}^{\infty} d_I^i(v),$$

$$d_F(v) = \sum_{i=1}^{\infty} d_F^i(v).$$

Proof. Since for each edge $e = (u,v) \in E_i \cap E_j$, we have $d_T^i(v)$ and $d_T^j(v)$ contain the term $T_{B_i}(u,v) = T_B(u,v)$, $T_{B_i}(u,v) = T_{B_i}(u,v)$ respectively. Hence, we conclude that $d_T(v) = d_T^i(v) + d_T^j(v) - T_B(u,v)$ whenever $E_i \cap E_j$ contains the only edge $e = (u,v)$. Hence if $E_i \cap E_j$ more than one edge, so we subtract them from the sum of the degree of truth membership vertex $v \in V$. Similarly, the degree of indeterminacy membership vertex, the degree of falsity membership are obtained.
If $E_i \cap E_j = \phi$ for all $1 \leq i, j$, so if $T_B(u,v) \neq 0$ in NF_{H_i}, then $T_B(u,v) = 0$ in NF_{H_j} this means that $T_B(u,v)$ will occur only one time. Hence, the result. □

5. Conclusions

The neutrosophic strong degree and total strong degree of the vertex of graphs were introduced with interesting properties on them. Furthermore, this paper discussed the concepts of strong edge, neutrosophic sum in NF – graph with suitable examples.

Conflicts of Interest: The authors declare no conflict of interest.

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