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ON THE POSITIVE SOLUTION OF MAX-TYPE DIFFERENCE EQUATION MODEL OF ORDER FIVE

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ABSTRACT. The goal of this work is to investigate the behavior of the solution of the following order five max-type difference equation

$$t_{i+1} = max\left\{\frac{C}{t_i}, t_{i-4}\right\}, \quad n \in N_0.$$

where the initial conditions t_{-4}, t_{-3}, t_{-2}, t_{-1}, t_0 are arbitrary positive real numbers and $(t_i)_{i=0}^{\infty}$ is a periodic sequence of period five.

1. INTRODUCTION

This is generally known: life is intimately connected to mathematics, and science is intrinsically linked to mathematics. One of the core ideas in connecting practical research with mathematics is the formation of equations. Discrete models are useful and effective instruments for assessing and solving a wide range of scientific and practical issues (see [1–4]). A recurrence relation that comprises the unknown function and its difference but not the derivative is called a difference equation. It is a strong modeling tool for describing real-world discrete-time systems. The difference equation model, for example, is widely used in algorithm analysis, control theory, computer science, biology, economics, and physics, among other subjects. Many fluid mathematical models can be transformed into discrete forms to improve numerical solutions, which can make the study process and findings easier to understand. As a result, difference equation models can be used to represent a wide range of natural and social phenomena (see [4–12]). The majority of the time, the condition parameters vary or change within a particular range, necessitating the adjustment of the equation model. Parameter values are subject to imperfections in an equation model that explains biological phenomena, for example, due to specific modifications [13]. The condition value of the qualifying product is a range rather than a fixed value in some quality control situations. Mozaffari proposed an intelligent framework for determining the exhaust gas temperature (T ex) and hydrocarbon emissions (HC raw) from an automobile engine during cold start operation in [14]. The conditional parameter values are unknown since the cold start operation is treated as a temporary and uncertain phenomenon, thus the adaptive neuro-fuzzy inference computation and fuzzy logic controller are employed in the work. As a result, equation models with manipulated variables and maximum or minimum type equation models

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are more important; they are commonly used in electronic engineering, computer, artificial intelligence, and science. In [15], Li investigated the stability of particle trajectories in particle swarms using a difference equation and Z-transform and discussed the effects of pBest, gBest, and randomness on particle trajectories. In [16], Zhu produced one-dimensional and two-dimensional time evolution equations models of nonlinear effect between pixels, based on his research on nonlinear impact of a digital image pixel grid. In [17], Chen proposed an excellent real-time autofocus technique using the discrete difference equation forecasting models. The proposed technique improves focusing speed. In [18], Fan used a difference equation model to examine a class of discrete SEIRS modeling techniques with general nonlinear occurrence and a discrete SEIRS epidemic model with standard occurrence. The condition that the sickness is permanent or that the model has a unique endemic equilibrium that is globally appealing is given by this equation model.

There has been a lot of interest in investigating the dynamics of the max-type difference equation model in few years (see [19–25]). These findings are not only useful in and of themselves, but they can also provide insight into their disparate equivalents. The study of max-type difference equations has recently come back a lot of interest. Difference equations of this type can be found in several forms in automatic control systems. At the start of their analysis of these equations, scientists concentrated on the behaviour of a few specific situations of the given difference equation.

In starting the investigation of these equations, professionals seem into the behavior of few specific instances of the following general equations of order $i \in N$.

$$t_i = max\left\{\frac{C_i^1}{t_{i-1}}, \frac{C_i^2}{t_{i-2}}, ..., \frac{C_i^n}{t_{i-n}}\right\}, i \in N_o \tag{1.1}$$

where $n \in N$, C_i^1, $i = 1, 2, ..., n$ are real sequence and the starting values $t_{-1}, t_{-2}, ..., t_{-n}$ are not zero, (see [26, 27] and the references in that). The steps in the review of the generalized max-type difference equation are as follows.:

$$t_i = max\ \left\{B_i^0, B_i^1\frac{t_{i-s_1}^{r_1}}{t_{i-t_!}^{p_1}}, B_i^2\frac{t_{i-s_2}^{r_2}}{t_{i-t_2}^{p_2}}, ..., B_i^i\frac{t_{i-s_i}^{r_i}}{t_{i-t_i}^{p_i}}\right\}, i \in N_o \tag{1.2}$$

where $i \in N$, s_i, t_i are natural numbers such that $s_1 < s_2 < ... < s_i$, $t_1 < t_2 < ... < t_i, r_k, p_i \in R$ and $B_n^n, n = 0, 1, 2, ...i$ are sequences of real numbers, was proposed by Stević in numerous discussions, for example in [34,35]

In [28], Simsek et al investigated the solutions of the max-type difference problem.

$$t_{i+1} = max\left\{t_{i-1}, \frac{1}{t_{i-1}}\right\} \tag{1.3}$$

In [29], Elabbasy et al. look at the max-type difference equation's solutions.

$$t_{i+1} = max\{t_{i-2}, \frac{1}{t_{i-2}}\} \tag{1.4}$$

In [30], The authors examined the second-order max-type difference equation below, which was inspired by some of stević's ideas.

$$t_{i+1} = max\left\{\frac{1}{t_i},\ Ct_{i-1}\right\}, i \in N_0. \tag{1.5}$$

Equation (1.5) is simple to work with because it can be transformed into one of the first-order difference equations below by modifying $t_i = t_i t_{i-1}$.

$$t_{i+1} = max\{1,\ Ct_i\} or t_{i+1} = min\{1, Ct_i\} \tag{1.6}$$

Using these equations, we can see that any solution of (5) is consequently periodic with period two under the condition $C = 1$.

Recently in [31], it was showed that every solution of the third-order max-type difference equation

$$t_{i+1} = max\left\{\frac{C}{t_i},\ t_{k-2}\right\}, i \in N_0. \tag{1.7}$$

Where the initial conditions are arbitrary non-zero real integers t_{-2}, t_{-1}, t_0, and $C \in R$, is consequently periodic with period three. It's hardly remarkable that all of (1.7) solutions are periodic (for an explanation see [32]).

Another max-type fourth-order difference equation is:

$$t_{i+1} = max\left\{\frac{C}{t_i},\ t_{k-3}\right\}, i \in N_0. \tag{1.8}$$

Where the initial conditions t_{-3}, t_{-2}, t_{-1}, t_0 are arbitrary non-zero real numbers and $C \in R$, is consequently periodic with period four.

We show that every well-defined solution of the fifth-order max-type difference equation is::

$$t_{i+1} = max\left\{\frac{C}{t_i},\ t_{i-4}\right\}, i \in N_0. \tag{1.9}$$

Where the initial conditions t_{-4} ,t_{-3}, t_{-2}, t_{-1}, t_0 are arbitrary non-zero real numbers and $C \in R$, is consequently periodic with period five. The parameter $C \in R_+ \cup \{0\}$, eventually becomes periodic with period five.

Definition 1.1. [33] A sequence $(t_i)_{i=-i}^{\infty}$ is said to be consequently periodic p if there exists an index $i_0 \in \{-i, ..., -1, 0, 1\}$ such that $t_{i+p} = t_i$ for all $i \geq n_0$. Notably, if $i_0 = -i$, then the sequence $(t_i)_{i=-i}^{\infty}$ is periodic with period p.

Remark 1.2. Note that if $C = 0$, then equation (1.9) becomes $t_{i+1} = t_{i-4}$, from which it follows that every solution has a period of five. s a result, we shall look at C_0 in the next section.

The following simple lemma, which is stated without a proof, will be used frequently in the following sections. (for related results see [30, 31]).

Lemma 1.3. *Let us assume that* $(t_i)_{i=-4}^{\infty}$ *is the solution of equation* (1.9) *and that is* $n_0 \in N_0 \cup$ *[-4, -3, -2, -1] such that*

$t_{n_0} = t_{n_0+5}$, $t_{n_0+1} = t_{n_0+6}$, $t_{n_0+2} = t_{n_0+7}$, $t_{n_0+3} = t_{n_0+8}$, $t_{n_0+4} = t_{n_0+9}$.

Then the solution is consequently periodic with period five.

2. Main results

When the parameter $C > 0$ is given, we provide a specific version of the solutions. In this part, we'll use the difference equation (1.9) to show that every solution to this equation is periodic with a period five.

There are the following (26) possibilities to evaluate based on the positivity of five initial values of equation (1.9).

(i) $t_{-4}, t_{-3}, t_{-2}, t_{-1}, t_0 > 0$
(ii) $t_{-4}, t_{-3}, t_{-2}, t_{-1}, t_0 < 0$
(iii) $t_0 < 0, t_{-4}, t_{-3}, t_{-2}, t_{-1} > 0$
(iv) $t_{-1} < 0, t_{-4}, t_{-3}, t_{-2}, t_0 > 0$
(v) $t_{-2} < 0, t_{-4}, t_{-3}, t_{-1}, t_0 > 0$
(vi) $t_{-3} < 0, t_{-4}, t_{-2}, t_{-1}, t_0 > 0$
(vii) $t_{-4} < 0, t_{-3}, t_{-2}, t_{-1}, t_0 > 0$
(viii) $t_0, t_{-1} < 0, t_{-4}, t_{-3}, t_{-2} > 0$

(ix) $t_0, t_{-2} < 0, t_{-4}, t_{-3}, t_{-1} > 0$
(x) $t_0, t_{-3} < 0, t_{-4}, t_{-2}, t_{-1} > 0$
(xi) $t_0, t_{-4} < 0, t_{-3}, t_{-2}, t_{-1} > 0$
(xii) $t_{-1}, t_{-2} < 0, t_{-4}, t_{-3}, t_0 > 0$
(xiii) $t_{-1}, t_{-3} < 0, t_{-4}, t_{-2}, t_0 > 0$
(xiv) $t_{-1}, t_{-4} < 0, t_{-3}, t_{-2}, t_0 > 0$
(xv) $t_{-2}, t_{-3} < 0, t_{-4}, t_{-1}, t_0 > 0$
(xvi) $t_{-2}, t_{-4} < 0, t_{-3}, t_{-1}, t_0 > 0$
(xvii) $t_{-3}, t_{-4} < 0, t_{-2}, t_{-1}, t_0 > 0$
(xviii) $t_0, t_{-1}, t_{-2} < 0, t_{-4}, t_{-3} > 0$
(xix) $t_0, t_{-1}, t_{-3} < 0, t_{-4}, t_{-2} > 0$
(xx) $t_0, t_{-1}, t_{-4} < 0, t_{-3}, t_{-2} > 0$
(xxi) $t_{-1}, t_{-2}, t_{-3} < 0, t_{-4}, t_0 > 0$
(xxii) $t_{-1}, t_{-2}, t_{-4} < 0, t_{-3}, t_0 > 0$
(xxiii) $t_{-2}, t_{-3}, t_{-4} < 0, t_{-1}, t_0 > 0$
(xxiv) $t_0, t_{-1}, t_{-2}, t_{-3} < 0, t_{-4} > 0$
(xxv) $t_0, t_{-1}, t_{-2}, t_{-4} < 0, t_{-3} > 0$
(xxvi) $t_{-1}, t_{-2}, t_{-3}, t_{-4} < 0, t_0 > 0$

Theorem 2.1. *Consider the difference equation with parameter $C > 0$ of equation* (1.9)*, then, at some initial points, every solution of equation* (1.9) *is periodic with period five.*

Proof. From equation (1.9), we have

$$t_1 \quad = \quad max\left\{\frac{C}{t_0}, t_{i-4}\right\}$$

We consider the following two cases:

Case C_1: If $t_0 t_{-4} \geq C$, then $t_1 = \frac{C}{t_0}$ and

$$t_2 = max\left\{\frac{C}{t_1}, t_{-3}\right\} = max\{t_0, t_{-3}\}$$

Now there exist three sub-cases.

C_{11}**:** If $t_0 \geq t_{-3}$, then $t_2 = t_0$ and

$$t_3 = max\left\{\frac{C}{t_2}, t_{-2}\right\} = max\left\{\frac{C}{t_0}, t_{-2}\right\}.$$

C_{111}**:** if $t_{-2} \geq \frac{C}{t_0}$, the $t_3 = t_{-2}$

$$t_4 = max\left\{\frac{C}{t_3}, t_{-1}\right\} = max\left\{\frac{C}{t_{-2}}, t_{-1}\right\}.$$

C_{1111}**:** if $t_{-1} \geq \frac{C}{t_{-2}}$, then $t_4 = t_{-1}$ and

$$t_5 = max\left\{\frac{C}{t_4}, t_0\right\} = max\left\{\frac{C}{t_{-1}}, t_0\right\} = t_0$$

$$t_6 = max\left\{\frac{C}{t_5}, t_1\right\} = max\left\{\frac{C}{t_0}, \frac{C}{t_0}\right\} = \frac{C}{t_0}$$

$$t_7 = max\left\{\frac{C}{t_6}, t_2\right\} = max\left\{t_0, t_0\right\} = t_0$$

$$t_8 = max\left\{\frac{C}{t_7}, t_3\right\} = max\left\{\frac{C}{t_0}, t_{-2}\right\} = t_{-2}$$

$$t_9 = max\left\{\frac{C}{t_8}, t_4\right\} = max\left\{\frac{C}{t_{-2}}, t_{-1}\right\} = t_{-1}$$

then we see that the solution is $\left\{t_{-4}, t_{-3}, t_{-2}, t_{-1}, t_0, \frac{C}{t_0}, t_0, t_{-2}, t_{-1}, t_0, \frac{C}{t_0}, t_0, t_{-2}, t_{-1}, \ldots\right\}$, and $(t_i)_{n=-4}^{\infty}$ is an eventually periodic solution with period five.

C_{1112}: $t_{-1} \leq \frac{C}{t_{-2}}$, then $t_4 = \frac{C}{t_{-2}}$ and

$$t_5 = max\left\{\frac{C}{t_4}, t_0\right\} = max\left\{t_{-2}, t_0\right\} = t_0$$

$$t_6 = max\left\{\frac{C}{t_5}, t_1\right\} = max\left\{\frac{C}{t_0}, \frac{C}{t_0}\right\} = \frac{C}{t_0}$$

$$t_7 = max\left\{\frac{C}{t_6}, t_2\right\} = max\{t_0, t_0\} = t_0$$

$$t_8 = max\left\{\frac{C}{t_7}, t_3\right\} = max\left\{\frac{C}{t_0}, t_{-2}\right\} = t_{-2}$$

(Because $t_{-2} \geq \frac{C}{t_0}$)

$$t_9 = max\left\{\frac{C}{t_8}, t_4\right\} = max\left\{\frac{C}{t_{-2}}, t_{-1}\right\} = \frac{C}{t_{-2}}$$

Then we see that the solution is $\left\{\frac{C}{t_{-2}}, t_0, \frac{C}{t_0}, t_0, t_{-2}, , \frac{C}{t_{-2}}, \ldots\right\}$, which is eventually periodic with period five.

C_{112}: $t_{-2} \leq \frac{C}{t_0}$, then $t_3 = \frac{C}{t_0}$ and

$$t_4 = max\left\{\frac{C}{t_3}, t_{-1}\right\} = max\left\{t_0, t_{-1}\right\}$$

C_{1121}: $t_0 \geq t_{-1}$, then $t_4 = t_0$ and

$$t_5 = max\left\{\frac{C}{t_4}, t_0\right\} = max\left\{\frac{C}{t_0}, t_0\right\} = t_0$$

$$t_6 = max\left\{\frac{C}{t_5}, t_1\right\} = max\left\{\frac{C}{t_0}, \frac{C}{t_0}\right\} = \frac{C}{t_0}$$

$$t_7 = max\left\{\frac{C}{t_6}, t_2\right\} = max\left\{t_0, t_0\right\} = t_0$$

$$t_8 = max\left\{\frac{C}{t_7}, t_3\right\} = max\left\{\frac{C}{t_0}, \frac{C}{t_0}\right\} = \frac{C}{t_0}$$

$$t_9 = max\left\{\frac{C}{t_8}, t_4\right\} = max\left\{t_0, t_{-1}\right\} = t_0,$$

then we see that the solution is $\left\{\frac{C}{t_0}, t_0, t_0, \frac{C}{t_0}, t_0, \frac{C}{t_0}, t_0, \ldots\right\}$, which is eventually periodic with period five.

C_{1122}: If $t_0 \leq t_{-1}$, then $t_4 = t_{-1}$ and

$$t_5 = max\left\{\frac{C}{t_4}, t_0\right\} = max\left\{\frac{C}{t_{-1}}, t_0\right\} = t_0$$

(Because $t_0 \geq \frac{C}{t_{-1}}$ in previous C)

$$t_6 = max\left\{\frac{C}{t_5}, t_1\right\} = max\left\{\frac{C}{t_0}, \frac{C}{t_0}\right\} = \frac{C}{t_0}$$

$$t_7 = max\left\{\frac{C}{t_6}, t_2\right\} = max\left\{t_0, t_0\right\} = t_0$$

$$t_8 = max\left\{\frac{C}{t_7}, t_3\right\} = max\left\{\frac{C}{t_0}, \frac{C}{t_0}\right\} = \frac{C}{t_0}$$

$$t_9 = max\left\{\frac{C}{t_8}, t_4\right\} = max\left\{t_0, t_{-1}\right\} = t_{-1}$$

Then we see that the solution is $\left\{t_{-1}, t_0, \frac{C}{t_0}, t_0, \frac{C}{t_0}, t_{-1}, \ldots\right\}$

C_{12}: If $t_0 \leq t_{-3}$, then $t_2 = t_{-3}$ and

$$t_3 = max\left\{\frac{C}{t_2}, t_{-2}\right\} = max\left\{\frac{C}{t_{-3}}, t_{-2}\right\}$$

C_{121}: If $t_{-2} \leq \frac{C}{t_{-3}}$, then $t_3 = \frac{C}{t_{-3}}$ and

$$t_4 = max\left\{\frac{C}{t_3}, t_{-1}\right\} = max\left\{t_{-3}, t_{-1}\right\}$$

C_{1211}: If $t_{-3} \leq t_{-1}$, then $t_4 = t_{-1}$ and

$$t_5 = max\left\{\frac{C}{t_4}, t_0\right\} = max\left\{\frac{C}{t_{-1}}, t_0\right\} = t_0$$

(Because $t_0 \geq \frac{C}{t_{-1}}$)

$$t_6 = max\left\{\frac{C}{t_5}, t_1\right\} = max\left\{\frac{C}{t_0}, \frac{C}{t_0}\right\} = \frac{C}{t_0}$$

$$t_7 = max\left\{\frac{C}{t_6}, t_2\right\} = max\left\{t_0, t_0\right\} = t_0$$

$$t_8 = max\left\{\frac{C}{t_7}, t_3\right\} = max\left\{\frac{C}{t_0}, \frac{C}{t_{-3}}\right\} = \frac{C}{t_{-3}}$$

(Because $\frac{t_0}{C} \geq \frac{t_{-3}}{C}, \frac{C}{t_0} \leq \frac{C}{t_{-3}}$)

$$t_9 = max\left\{\frac{C}{t_8}, t_4\right\} = max\left\{t_{-3}, t_{-1}\right\} = t_{-1}$$

Then we see that the solution is $\left\{t_{-1}, t_0, \frac{C}{t_0}, t_0, \frac{C}{t_{-3}}, t_{-1}, \ldots\right\}$, which is eventually periodic with period five.

C_{1212}: If $t_{-3} \geq t_{-1}$, then $t_4 = t_{-3}$ and

$$t_5 = max\left\{\frac{C}{t_4}, t_0\right\} = max\left\{\frac{C}{t_{-3}}, t_0\right\} = t_0$$

$$t_6 = max\left\{\frac{C}{t_5}, t_1\right\} = max\left\{\frac{C}{t_0}, \frac{C}{t_0}\right\} = \frac{C}{t_0}$$

$$t_7 = max\left\{\frac{C}{t_6}, t_2\right\} = max\left\{t_0, t_{-1}\right\} = t_{-1}$$

Because $(t_{-1} \geq t_0)$

$$t_8 = max\left\{\frac{C}{t_7}, t_3\right\} = max\left\{\frac{C}{t_{-1}}, \frac{C}{t_{-3}}\right\} = \frac{C}{t_{-3}}$$

$$t_9 = max\left\{\frac{C}{t_8}, t_4\right\} = max\left\{t_{-3}, t_{-3}\right\} = t_{-3}$$

Then we see that the solution is $\left\{t_{-3}, t_0, \frac{C}{t_0}, t_{-1}, \frac{C}{t_{-3}}, t_{-3}, \ldots\right\}$, which is eventually periodic with period five.

C_{122}: If $t_{-2} \geq \frac{C}{t_{-3}}$, then $t_3 = t_{-2}$ and

$$t_4 = max\left\{\frac{C}{t_3}, t_{-1}\right\} = max\left\{\frac{C}{t_{-2}}, t_{-1}\right\} = t_{-1}$$

$$t_5 = max\left\{\frac{C}{t_4}, t_0\right\} = max\left\{\frac{C}{t_{-1}}, t_0\right\} = t_0$$

$$t_6 = max\left\{\frac{C}{t_5}, t_1\right\} = max\left\{\frac{C}{t_0}, \frac{C}{t_0}\right\} = \frac{C}{t_0}$$

$$t_7 = max\left\{\frac{C}{t_6}, t_2\right\} = max\left\{t_0, t_{-3}\right\}$$

C_{1221}: If $t_{-3} \geq t_0$, then $t_7 = t_{-3}$ and

$$t_8 = max\left\{\frac{C}{t_7}, t_3\right\} = max\left\{\frac{C}{t_{-3}}, t_{-2}\right\} = t_{-2}$$

$$t_9 = max\left\{\frac{C}{t_8}, t_4\right\} = max\left\{\frac{C}{t_{-2}}, t_{-1}\right\} = t_{-1}$$

Then we see that the solution is $\left\{t_{-2}, t_{-1}, t_0, \frac{C}{t_0}, t_{-3}, t_{-2}, t_{-1}, \ldots\right\}$, which is eventually periodic with period five.

C_{1222}: If $t_{-3} \leq t_0$, then $t_7 = t_0$ and

$$t_7 = max\left\{\frac{C}{t_6}, t_2\right\} = max\left\{\frac{C}{t_0}, t_{-3}\right\} = t_{-3}$$

$$t_8 = max\left\{\frac{C}{t_7}, t_3\right\} = max\left\{\frac{C}{t_{-3}}, t_{-2}\right\} = t_{-2}$$

$$t_9 = max\left\{\frac{C}{t_8}, t_4\right\} = max\left\{\frac{C}{t_{-2}}, t_{-1}\right\} = t_{-1}$$

$$t_{10} = max\left\{\frac{C}{t_9}, t_5\right\} = max\left\{\frac{C}{t_{-1}}, t_0\right\} = t_0$$

$$t_{11} = max\left\{\frac{C}{t_{10}}, t_6\right\} = max\left\{\frac{C}{t_0}, \frac{C}{t_0}\right\} = \frac{C}{t_0}$$

$$t_{12} = max\left\{\frac{C}{t_{11}}, t_7\right\} = max\left\{t_0, t_0\right\} = t_0$$

Then we see that the solution is $\left\{t_0, t_{-3}t_{-2}, t_{-1}, t_0, \frac{C}{t_0}, t_0, \ldots\right\}$, which is eventually periodic with period five.

Case C_2: If $t_{-4} \geq \frac{C}{t_0}$, then $t_1 = t_{-4}$ and

$$t_2 = max\left\{\frac{C}{t_1}, t_{-3}\right\} = max\left\{\frac{C}{t_{-4}}, t_{-3}\right\}$$

C_{12}: If $t_{-4}t_{-3} \leq C$, then $t_2 = \frac{C}{t_{-4}}$ and

$$t_3 = max\left\{\frac{C}{t_2}, t_{-2}\right\} = max\left\{t_{-4}, t_{-2}\right\}$$

C_{211}: If $t_{-2} \geq t_{-4}$, then $t_3 = t_{-2}$ and

$$t_4 = max\left\{\frac{C}{t_3}, t_{-1}\right\} = max\left\{\frac{C}{t_{-2}}, t_{-1}\right\} = t_{-1}$$

C_{2111}: If $\frac{C}{t_{-2}} \geq t_{-1}$, then $t_4 = \frac{C}{t_{-2}}$ and

$$t_5 = max\left\{\frac{C}{t_4}, t_0\right\} = max\left\{t_{-2}, t_0\right\}$$

C_{21111}: If $t_{-2} \geq t_0$, then $t_5 = t_{-2}$ and

$$t_6 = max\left\{\frac{C}{t_5}, t_1\right\} = max\left\{\frac{C}{t_{-2}}, t_{-4}\right\} = t_{-4}$$

$$t_7 = max\left\{\frac{C}{t_6}, t_2\right\} = max\left\{\frac{C}{t_{-4}}, \frac{C}{t_{-4}}\right\} = \frac{C}{t_{-4}}$$

$$t_8 = max\left\{\frac{C}{t_7}, t_3\right\} = max\left\{t_{-4}, t_{-2}\right\} = t_{-2}$$

$$t_9 = max\left\{\frac{C}{t_8}, t_4\right\} = max\left\{\frac{C}{t_{-2}}, \frac{C}{t_{-2}}\right\} = \frac{C}{t_{-2}}$$

$$t_{10} = max\left\{\frac{C}{t_9}, t_5\right\} = max\left\{t_{-2}, t_{-2}\right\} = t_{-2}$$

Then we see that the solution is $\left\{t_{-2}, t_{-4}, \frac{C}{t_{-4}}, t_{-2}, \frac{C}{t_{-2}}, t_{-2}, \ldots\right\}$, which is eventually periodic with period five.

C_{21112}: If $t_0 \geq t_{-2}$, then $t_5 = t_0$ and

$$t_6 = max\left\{\frac{C}{t_5}, t_1\right\} = max\left\{\frac{C}{t_0}, t_{-4}\right\} = t_{-4}$$

$$t_7 = max\left\{\frac{C}{t_6}, t_2\right\} = max\left\{\frac{C}{t_{-4}}, \frac{C}{t_{-4}}\right\} = \frac{C}{t_{-4}}$$

$$t_8 = max\left\{\frac{C}{t_7}, t_3\right\} = max\left\{t_{-4}, t_{-2}\right\} = t_{-2}$$

$$t_9 = max\left\{\frac{C}{t_8}, t_4\right\} = max\left\{\frac{C}{t_{-2}}, \frac{C}{t_{-2}}\right\} = \frac{C}{t_{-2}}$$

$$t_{10} = max\left\{\frac{C}{t_9}, t_5\right\} = max\left\{t_{-2}, t_0\right\} = t_0$$

Then we see that the solution is $\left\{t_0, t_{-4}, \frac{C}{t_{-4}}, t_{-2}, \frac{C}{t_{-2}}, t_0, \ldots\right\}$, which is eventually periodic with period five.

C_{2112}: If $\frac{C}{t_{-2}} \leq t_{-1}$, then $t_4 = t_{-1}$ and

$$t_5 = max\left\{\frac{C}{t_4}, t_0\right\} = max\left\{\frac{C}{t_{-1}}, t_0\right\} = t_0$$

$$t_6 = max\left\{\frac{C}{t_5}, t_1\right\} = max\left\{\frac{C}{t_0}, t_{-4}\right\} = t_{-4}$$

$$t_7 = max\left\{\frac{C}{t_6}, t_2\right\} = max\left\{\frac{C}{t_{-4}}, \frac{C}{t_{-4}}\right\} = \frac{C}{t_{-4}}$$

$$t_8 = max\left\{\frac{C}{t_7}, t_3\right\} = max\left\{t_{-4}, t_{-2}\right\} = t_{-2}$$

$$t_9 = max\left\{\frac{C}{t_8}, t_4\right\} = max\left\{\frac{C}{t_{-2}}, t_{-1}\right\} = t_{-1}$$

Then we see that the solution is $\left\{t_{-1}, t_0, t_{-4}, \frac{C}{t_{-4}}, t_{-2}, t_{-1}, \ldots\right\}$, which is eventually periodic with period five.

C_{21121}: If $t_0 \leq t_{-2}$, then $t_5 = t_{-2}$ and

$$t_6 = max\left\{\frac{C}{t_5}, t_1\right\} = max\left\{\frac{C}{t_{-2}}, t_{-4}\right\} = t_{-4}$$

(Because $t_{-4} \geq \frac{C}{t_0} \geq \frac{C}{t_{-1}} \geq \frac{C}{t_{-2}} \geq \frac{C}{t_{-3}} \geq \frac{C}{t_{-4}}$)

$$t_7 = max\left\{\frac{C}{t_6}, t_2\right\} = max\left\{\frac{C}{t_{-4}}, \frac{C}{t_{-4}}\right\} = \frac{C}{t_{-4}}$$

$$t_8 = max\left\{\frac{C}{t_7}, t_3\right\} = max\left\{t_{-4}, t_{-2}\right\} = t_{-2}$$

$$t_9 = max\left\{\frac{C}{t_8}, t_4\right\} = max\left\{\frac{C}{t_{-2}}, t_{-1}\right\} = t_{-1}$$

$$t_{10} = max\left\{\frac{C}{t_9}, t_5\right\} = max\left\{\frac{C}{t_{-1}}, t_{-2}\right\} = t_{-2}$$

Then we see that the solution is $\left\{t_{-2}, t_{-4}, \frac{C}{t_{-4}}, t_{-2}, t_{-1}, t_{-2}, \ldots\right\}$, which is eventually periodic with period five.

C_{21122}: If $t_0 \geq t_{-2}$, then $t_5 = t_0$ and

$$t_6 = max\left\{\frac{C}{t_5}, t_1\right\} = max\left\{\frac{C}{t_0}, t_{-4}\right\} = t_{-4}$$

$$t_7 = max\left\{\frac{C}{t_6}, t_2\right\} = max\left\{\frac{C}{t_{-4}}, \frac{C}{t_{-4}}\right\} = \frac{C}{t_{-4}}$$

$$t_8 = max\left\{\frac{C}{t_7}, t_3\right\} = max\left\{t_{-4}, t_{-2}\right\} = t_{-2}$$

$$t_9 = max\left\{\frac{C}{t_8}, t_4\right\} = max\left\{\frac{C}{t_{-2}}, t_{-1}\right\} = t_{-1}$$

$$t_{10} = max\left\{\frac{C}{t_9}, t_5\right\} = max\left\{\frac{C}{t_{-1}}, t_0\right\} = t_0$$

Then we see that the solution is $\left\{t_0, t_{-4}, \frac{C}{t_{-4}}, t_{-2}, t_{-1}, t_0, \dots\right\}$, which is eventually periodic with period five.

C_{212}: If $t_{-2} \le t_{-4}$, then $t_3 = t_{-4}$ and

$$t_4 = max\left\{\frac{C}{t_3}, t_{-1}\right\} = max\left\{\frac{C}{t_{-4}}, t_{-1}\right\} = t_{-1}$$

$$t_5 = max\left\{\frac{C}{t_4}, t_0\right\} = max\left\{\frac{C}{t_{-1}}, t_0\right\} = t_0$$

$$t_6 = max\left\{\frac{C}{t_5}, t_1\right\} = max\left\{\frac{C}{t_0}, t_{-4}\right\} = t_{-4}$$

$$t_7 = max\left\{\frac{C}{t_6}, t_2\right\} = max\left\{\frac{C}{t_{-4}}, \frac{C}{t_{-4}}\right\} = \frac{C}{t_{-4}}$$

$$t_8 = max\left\{\frac{C}{t_7}, t_3\right\} = max\left\{t_{-4}, t_{-4}\right\} = t_{-4}$$

$$t_9 = max\left\{\frac{C}{t_8}, t_4\right\} = max\left\{\frac{C}{t_{-4}}, t_{-1}\right\} = t_{-1}$$

Then we see that the solution is $\left\{t_{-4}, t_{-1}, t_0, \frac{C}{t_{-4}}, t_{-4}, t_{-1}, \dots\right\}$, which is eventually periodic with period five.

C_{22}: If $t_{-4}t_{-3} \ge C$, then $t_2 = \frac{C}{t_{-3}}$ and

$$t_3 = max\left\{\frac{C}{t_2}, t_{-2}\right\} = max\left\{t_{-3}, t_{-2}\right\}$$

C_{221}: If $t_{-2} \le \frac{C}{t_{-3}}$, then $t_3 = \frac{C}{t_{-3}}$ and

$$t_4 = max\left\{\frac{C}{t_3}, t_{-1}\right\} = max\left\{t_{-2}, t_{-1}\right\}$$

C_{2211}: If $t_{-2} \ge t_{-4}$, then $t_3 = t_{-2}$ and

$$t_4 = max\left\{\frac{C}{t_3}, t_{-1}\right\} = max\left\{\frac{C}{t_{-2}}, t_{-1}\right\} = t_{-1}$$

$$t_5 = max\left\{\frac{C}{t_4}, t_0\right\} = max\left\{\frac{C}{t_{-1}}, t_0\right\} = t_0$$

$$t_6 = max\left\{\frac{C}{t_5}, t_1\right\} = max\left\{\frac{C}{t_0}, \frac{C}{t_{-4}}\right\} = \frac{C}{t_{-4}}$$

$$t_7 = max\left\{\frac{C}{t_6}, t_2\right\} = max\left\{t_{-4}, \frac{C}{t_{-4}}\right\} = t_{-4}$$

$$t_8 = max\left\{\frac{C}{t_7}, t_3\right\} = max\left\{\frac{C}{t_{-4}}, t_{-2}\right\} = t_{-2}$$

$$t_9 = max\left\{\frac{C}{t_8}, t_4\right\} = max\left\{\frac{C}{t_{-4}}, t_{-1}\right\} = t_{-1}$$

Then we see that the solution is $\left\{t_{-2}, t_{-1}, t_0, \frac{C}{t_{-4}}, t_{-4}, t_{-2}, t_{-1}, \dots\right\}$, which is eventually periodic with period five.

C_{2212}**:** If $t_{-2} \leq t_{-4}$, then $t_3 = t_{-4}$ and

$$t_4 = max\left\{\frac{C}{t_3}, t_{-1}\right\} = max\left\{\frac{C}{t_{-4}}, t_{-1}\right\} = t_{-1}$$

$$t_5 = max\left\{\frac{C}{t_4}, t_0\right\} = max\left\{\frac{C}{t_{-1}}, t_0\right\} = t_0$$

$$t_6 = max\left\{\frac{C}{t_5}, t_1\right\} = max\left\{\frac{C}{t_0}, t_{-4}\right\} = t_{-4}$$

$$t_7 = max\left\{\frac{C}{t_6}, t_2\right\} = max\left\{\frac{C}{t_{-4}}, \frac{C}{t_{-4}}\right\} = \frac{C}{t_{-4}}$$

$$t_8 = max\left\{\frac{C}{t_7}, t_3\right\} = max\left\{t_{-4}, t_{-4}\right\} = t_{-4}$$

$$t_9 = max\left\{\frac{C}{t_8}, t_4\right\} = max\left\{\frac{C}{t_{-4}}, t_{-1}\right\} = t_{-1}$$

Then we see that the solution is $\left\{t_{-4}, t_{-1}, t_0, t_{-4}, \frac{C}{t_{-4}}, t_{-4}, t_{-1}, \ldots\right\}$, which is eventually periodic with period five.

C_{222}**:** If $t_{-2} \geq \frac{C}{t_{-4}}$, then $t_3 = t_{-2}$ and

$$t_4 = max\left\{\frac{C}{t_3}, t_{-1}\right\} = max\left\{\frac{C}{t_{-2}}, t_{-1}\right\}$$

C_{2221}**:** If $\frac{C}{t_{-2}} \leq t_{-1}$, then $t_4 = t_{-1}$ and

$$t_5 = max\left\{\frac{C}{t_4}, t_0\right\} = max\left\{\frac{C}{t_{-1}}, t_0\right\} = t_0$$

$$t_6 = max\left\{\frac{C}{t_5}, t_1\right\} = max\left\{\frac{C}{t_0}, t_{-4}\right\}$$

C_{22211}: If $\frac{C}{t_0} \leq t_{-4}$, then $t_6 = t_{-4}$ and

$$t_7 = max\left\{\frac{C}{t_6}, t_2\right\} = max\left\{\frac{C}{t_{-4}}, \frac{C}{t_{-4}}\right\} = \frac{C}{t_{-4}}$$

$$t_8 = max\left\{\frac{C}{t_7}, t_3\right\} = max\left\{t_{-4}, t_{-4}\right\} = t_{-4}$$

$$t_9 = max\left\{\frac{C}{t_8}, t_4\right\} = max\left\{\frac{C}{t_{-4}}, t_{-1}\right\} = t_{-1}$$

$$t_{10} = max\left\{\frac{C}{t_9}, t_5\right\} = max\left\{\frac{C}{t_{-1}}, t_0\right\} = t_0$$

Then we see that the solution is $\left\{t_{-4}, \frac{C}{t_{-4}}, t_{-4}, t_{-1}, t_0, t_{-4}, \ldots\right\}$, which is eventually periodic with period five.

C_{22212}**:** If $\frac{C}{t_{-1}} \geq t_0$, then $t_6 = \frac{C}{t_{-1}}$ and

$$t_7 = max\left\{\frac{C}{t_6}, t_2\right\} = max\left\{t_{-1}, \frac{C}{t_{-3}}\right\} = \frac{C}{t_{-3}}$$

$$t_8 = max\left\{\frac{C}{t_7}, t_3\right\} = max\left\{t_{-3}, t_{-2}\right\} = t_{-2}$$

$$t_9 = max\left\{\frac{C}{t_8}, t_4\right\} = max\left\{\frac{C}{t_{-2}}, t_{-1}\right\} = \frac{C}{t_{-2}}$$

$$t_{10} = max\left\{\frac{C}{t_9}, t_5\right\} = max\left\{t_{-2}, t_0\right\} = t_0$$

$$t_{11} = max\left\{\frac{C}{t_{10}}, t_6\right\} = max\left\{\frac{C}{t_0}, \frac{C}{t_{-1}}\right\} = \frac{C}{t_{-1}}$$

Then we see that the solution is $\left\{\frac{C}{t_{-1}}, \frac{C}{t_{-3}}, t_{-2}, \frac{C}{t_{-2}}, t_0, \frac{C}{t_{-1}}, \ldots\right\}$, which is eventually periodic with period five.

C_{2222}: If $\frac{C}{t_{-2}} \geq t_{-1}$, then $t_4 = \frac{C}{t_{-2}}$ and

$$t_5 = max\left\{\frac{C}{t_4}, t_0\right\} = max\left\{t_{-2}, t_0\right\} = t_0$$

$$t_6 = max\left\{\frac{C}{t_5}, t_1\right\} = max\left\{\frac{C}{t_0}, t_{-4}\right\}$$

C_{22221}: If $\frac{C}{t_0} \geq t_{-4}$, then $t_6 = \frac{C}{t_0}$ and

$$t_7 = max\left\{\frac{C}{t_6}, t_2\right\} = max\left\{t_0, \frac{C}{t_{-3}}\right\} = \frac{C}{t_{-3}}$$

$$t_8 = max\left\{\frac{C}{t_7}, t_3\right\} = max\left\{t_{-3}, t_{-2}\right\} = t_{-2}$$

$$t_9 = max\left\{\frac{C}{t_8}, t_4\right\} = max\left\{\frac{C}{t_{-2}}, \frac{C}{t_{-2}}\right\} = \frac{C}{t_{-2}}$$

$$t_{10} = max\left\{\frac{C}{t_9}, t_5\right\} = max\left\{t_{-2}, t_0\right\} = t_0$$

$$t_{11} = max\left\{\frac{C}{t_{10}}, t_6\right\} = max\left\{\frac{C}{t_0}, \frac{C}{t_0}\right\} = \frac{C}{t_0}$$

Then we see that the solution is $\left\{\frac{C}{t_0}, \frac{C}{t_{-3}}, t_{-2}, \frac{C}{t_{-2}}, t_0, \frac{C}{t_0}, \ldots\right\}$, which is eventually periodic with period five.

C_{22222}: If $\frac{C}{t_0} \leq t_{-4}$, then $t_6 = t_{-4}$ and

$$t_7 = max\left\{\frac{C}{t_6}, t_2\right\} = max\left\{\frac{C}{t_{-4}}, \frac{C}{t_{-3}}\right\} = \frac{C}{t_{-4}}$$

$$t_8 = max\left\{\frac{C}{t_7}, t_3\right\} = max\left\{t_{-4}, t_{-2}\right\} = t_{-2}$$

$$t_9 = max\left\{\frac{C}{t_8}, t_4\right\} = max\left\{\frac{C}{t_{-2}}, \frac{C}{t_{-2}}\right\} = \frac{C}{t_{-2}}$$

$$t_{10} = max\left\{\frac{C}{t_9}, t_5\right\} = max\left\{t_{-2}, t_0\right\} = t_0$$

$$t_{11} = max\left\{\frac{C}{t_{10}}, t_6\right\} = max\left\{\frac{C}{t_0}, t_{-4}\right\} = t_{-4}$$

Then we see that the solution is $\left\{t_{-4}, \frac{C}{t_{-4}}, t_{-2}, \frac{C}{t_{-2}}, t_0, t_{-4}, \ldots\right\}$, which is eventually periodic with period five.

□

Theorem 2.2. *Assume that the parameter $C > 0$ and all the initial values are negative $t_{-4},\ t_{-3},\ t_{-2},\ t_{-1},\ t_0 < 0$. Then every solution of equation* (1.9) *is eventually periodic with period five.*

Proof. Since $t_{-4} < 0$, $t_{-3} < 0$, $t_{-2} < 0$, $t_{-1} < 0$, $t_0 < 0$,and $C > 0$, by induction we have $t_i < 0$ for each

$$t_{i+1} = (min\{t_i, \frac{1}{t_{i-4}}\})^{-1} = max\{\frac{1}{t_i}, t_{i-4}\}$$

Where $t_i > 0$ for every $i \geq -4$. Hence , the result. □

3. Conclusion

We finish this paper with some possible directions for future development. This article does not explain the case where parameter A is negative. We are able to verify that in many sub-cases, all the solutions of equation (1.9) are finally periodic with period five using comparable methods. We decided not to publish any result when $C < 0$ because there are too many sub-cases and calculations without any new ideas. when $C < 0$, We conjecture that every solution of equation (1.9) is eventually periodic with the same period five. What is even more interesting is figuring out a way to prove the conjecture in a reasonable amount of time without resorting to tiresome calculations like those in Theorem (2.1).

3.1. Periodicity.

We should also point out that we proved in [32] that every solution of the equation

$$t_{i+1} = max\left\{\frac{C}{t_{k-j}}, t_{k-l}\right\}, i \in N_0$$

Where $j, l \in N_0$, is periodic. It is of some interest to find the minimal period of the equation, as well as to get a general result concerning this problem.

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