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VARIATIONAL ITERATION METHOD FOR APPROXIMATE SOLUTION OF COVID-19 MODEL

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ABSTRACT. In this manuscript, we investigate a mathematical model of COVID-19 for approximate solution using variation iteration method (VIM). With the help of said technique we developed an algorithm to compute series type solution to the considered problem. Then using some real values for parameters and initial data we compute few terms approximate solutions. With the help of MMATLAB, we also plot our approximate solutions for different compartments graphically.

1. INTRODUCTION

The entire globe face a contagious pandemic called Novel Corona Virus, which is considered to spread from Wuhan, China. In January 2020, it was diagnosed in a single infected person and later identified in more patients [7]. There is assumption that this COVID-19 is from zoonotic origin, as first 425 confirmed cases which 55 percent of the total affected people were linked to the marketplace [8], this is the reason that it was assumed that wholesale market of live animal and seafood in Wuhan, and Huanan Seafood Market were the origin of this pandemic. 96 percent of similarity found between the bat coronaviruses and COVID-19 genetic sequences [9]. This pandemic is the third in recent past, the first was SARS-COV- severe acute respiratory syndrome coronavirus, this virus was outbreak 37 countries in 2002, whereas MERS-COV-Middle East respiratory syndrome coronavirus spread among 27 countries in 2012. The generic symptoms of infection in all three pandemics-COVID- 19, MERS-COV, and SARS-COV [10] were found to be similar which includes: breathing difficulty, fever, fatigue, whereas in severe cases lung infiltration. There are other symptoms other than respiratory includes diarrhea, nausea, and vomiting [11, 12]. During the late January 2020, confirmed cases increased in China and the coronavirus was spread to Republic of Korea, Japan, U.S., and Thailand [13, 14]. In order, to control the spread of the COVID-19, the government of China imposed lock down in Wuhan and more than a dozen cities in January 2020. These steps enable the government to restrict movement of around 50 million people in central China which was one of the largest quarantines in the history of mankind. In February 2020, approximately more than 40,000 individuals contracted the coronavirus [13]. In late January, the World Health Organization (WHO) formally declared the outbreak of novel coronavirus a Global Public Health Emergency of International Concern [15]. Wild animals for instance civets, bats etc [9] are the main source of the coronavirus however, many factors complicate the spread of the novel coronavirus which are difficult to control it. The clinical evidence recommends 2 to 14 days as incubation period of the COVID - 19 though the infected persons may not be aware of their infection

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or may be asymptomatic but non infected people can contract coronavirus from them or they can transmit the coronavirus to other person [16]. Moreover, this is a new virus for which fully approved antiviral drugs or vaccines are not available to treat though many countries developed vaccines and got approved in emergencies but its effectives yet to be validated by the scientific evidence. Therefore, countries around the world along with vaccination drive also encourage to isolate the infected individual to control the spread of the disease.Several modeling studies have already been conducted for the novel coronavirus pandemic. Majority of these models have brought-out the significant role of the direct, human-to-human transmission pathway in this pandemic [22], the study further shows that majority of the infected individuals did not have any contact with the marketplaces in Wuhan, however, the COVID-19 infection has increased rapidly across China's provinces and around the world. As a matter of fact, the infected persons are asymptomatic for up to 10-14 days but they can be a main source of the spread of COVID-19 and transmit the disease to other individuals through in-person contact. Furthermore, the models studied so far do not consider the role of the environment in the transmission of COVID-19. For instance, it is described that environmental samples taken from the Huanan Seafood Market areas have resulted in positive for the COVID-19 virus [10], which shows that the virus could be spread through the environmental reservoir. The infected individual of COVID-19 could possibly spread the virus into the environment through their respiratory droplets, because of their sneeze or cough, which then transmit the COVID-19 viruses to other persons with close contact of the same area. Also, there might be a probability that the virus could survive in the environment for many days, which may increase the risk of contamination via surfaces and fomites [23, 24]. Such environmental survival was confirmed for SARS-COV [25]. The most updated and recent study, based on the review of 22 types of coronaviruses, suggests that coronaviruses for instance MERS-COV, SARS-COV, and endemic human coronaviruses can survive on inanimate surfaces like metal, glass, or plastic for up to 9 days [26], which provide powerful evidence for the pathogen's environmental survival. In addition, the COVID-19 virus was also found in the stool of some infected persons [11], which means when the aquatic environment gets polluted then the chances of the outbreak and spread of COVID-19 disease increased.

2. Model formulation

We divide the total human population into four compartments: the non-infected (denoted by S), the exposed (denoted by E), the infected (denoted by I), the recovered (denoted by R). We considered that the infected individuals have fully developed disease symptoms and can infect others. And the exposed individuals can also infect others but still don't have any symptoms. We introduce the following model to describe the transmission dynamics of the COVID-19 epidemic: The Model is given as

$$\begin{cases} \dfrac{d}{dt}S(t) = \gamma - k(1+\alpha I(t))S(t)I(t) - \varepsilon S(t), \\ \dfrac{d}{dt}E(t) = k(1+\alpha I(t))S(t)I(t) - (\varepsilon+\delta)E(t), \\ \dfrac{d}{dt}I(t) = \eta + \delta E(t) - (\upsilon + \varepsilon + \beta)I(t), \\ \dfrac{d}{dt}R(t) = \beta I(t) - \varepsilon R(t). \\ S(0) = S_0, E(0) = E_0, I(0) = I_0, R(0) = R_0. \end{cases} \tag{1}$$

3. Background materials

To apply the VIM for a linear and nonlinear problems consider the following differential equation

$$Lw + Nw = g(t), \tag{2}$$

where L and N are linear and nonlinear operators respectively, and $g(t)$ is the source inhomogeneous term. According to the variational iteration method we can construct a correct functional as follows:

$$w_{n+1}(x) = w_n(x) + \int_0^x \lambda(\xi)(Lw + N\tilde{w} - g(\xi))d\xi, \tag{3}$$

where λ is a general Lagrange multiplier, which can be identified optimally via the variational theory, the subscript n denotes the nth approximation,$\tilde{w}_n$ is considered as a restricted variation i.e $\delta\tilde{w}_n = 0$. For linear problems, its exact solution can be obtained by only one iteration step due to the fact that the Lagrange multiplier can be exactly identified.

4. Main work

Here we find the approximate solution of the proposed model using VIM. The correction functional for each equation in the model (1) are as follow

$$\begin{cases} \mathbb{S}_{n+1}(t) = \mathbb{S}_n(t) + \int_0^t \lambda(\varphi)(\mathbb{S}'_n(\varphi) - \gamma - k(1+\alpha\mathbb{I}_n(t))\mathbb{S}_n(t)\mathbb{I}_n(t) + \varepsilon\mathbb{S}_n(t))d\varphi, \\ \mathbb{E}_{n+1}(t) = \mathbb{E}_n(t) + \int_0^t \lambda(\varphi)(\mathbb{E}'_n(\varphi) - k(1+\alpha\mathbb{I}_n(t))\mathbb{S}_n(t)\mathbb{I}_n(t) + (\varepsilon+\delta)\mathbb{E}_n(t))d\varphi, \\ \mathbb{I}_{n+1}(t) = \mathbb{I}_n(t) + \int_0^t \lambda(\varphi)(\mathbb{I}'_n(\varphi) - \eta - \delta\mathbb{E}_n(t) + (\upsilon+\varepsilon+\beta)\mathbb{I}_n(t))d\varphi, \\ \mathbb{R}_{n+1}(t) = \mathbb{R}_n(t) + \int_0^t \lambda(\varphi)(\mathbb{R}'_n(\varphi) - \beta\mathbb{I}_n(t)(t) + \varepsilon\mathbb{R}_n(t))d\varphi, \end{cases} \tag{4}$$

In each equations we put

$$\lambda(\varphi) = -1.$$

because in each equations first derivative are involved. Now the above system of equations become

$$\begin{cases} \mathbb{S}_{n+1}(t) = \mathbb{S}_n(t) - \int_0^t (\mathbb{S}'_n(\varphi) - \gamma - k(1+\alpha\mathbb{I}_n(t))\mathbb{S}_n(t)\mathbb{I}_n(t) + \varepsilon\mathbb{S}_n(t))d\varphi, \\ \mathbb{E}_{n+1}(t) = \mathbb{E}_n(t) - \int_0^t (\mathbb{E}'_n(\varphi) - k(1+\alpha\mathbb{I}_n(t))\mathbb{S}_n(t)\mathbb{I}_n(t) + (\varepsilon+\delta)\mathbb{E}_n(t))d\varphi, \\ \mathbb{I}_{n+1}(t) = \mathbb{I}_n(t) - \int_0^t (\mathbb{I}'_n(\varphi) - \eta - \delta\mathbb{E}_n(t) + (\upsilon+\varepsilon+\beta)\mathbb{I}_n(t))d\varphi, \\ \mathbb{R}_{n+1}(t) = \mathbb{R}_n(t) - \int_0^t (\mathbb{R}'_n(\varphi) - \beta\mathbb{I}_n(t)(t) + \varepsilon\mathbb{R}_n(t))d\varphi, \end{cases} \tag{5}$$

Here onward we discuss some cases to compute approximate solution up to few terms.

CASE 1 For n=0. We get the following equations from system of equations (5) after computation,

$$\begin{cases} \mathbb{S}_1(t) = \mathbb{S}_0 - (-\gamma - k(1+\alpha\mathbb{I}_0)\mathbb{S}_0\mathbb{I}_0 + \varepsilon\mathbb{S}_0)t, \\ \mathbb{E}_1(t) = \mathbb{E}_0 - (-k(1+\alpha\mathbb{I}_0)\mathbb{S}_0\mathbb{I}_0 + (\varepsilon+\delta)\mathbb{E}_0)t, \\ \mathbb{I}_1(t) = \mathbb{I}_0 - (-\eta + \delta\mathbb{E}_0 + (\upsilon+\varepsilon+\beta)\mathbb{I}_0)t, \\ \mathbb{R}_1(t) = \mathbb{R}_0 - (-\beta\mathbb{I}_0 + \varepsilon\mathbb{R}_0)t. \end{cases} \tag{6}$$

CASE 1 For n=1. From the system of equations (5), by putting the values of $\mathbb{S}_1(t)$, $\mathbb{E}_1(t)$, $\mathbb{I}_1(t)$ and $\mathbb{R}_1(t)$ we get the following system of equations

$$\begin{aligned}
&\mathbb{S}_2(t) = \mathbb{S}_0 - [-\wedge + \beta_0\mathbb{S}_0\mathbb{E}_0 + \beta_1\mathbb{S}_0\mathbb{I}_0 + \beta_2\mathbb{S}_0\mathbb{V}_0 + \mu_0\mathbb{S}_0]t \\
&+ [-(\beta_0)^2\mathbb{S}_0^2\mathbb{E}_0 - \beta_0\beta_1\mathbb{S}_0^2\mathbb{I}_0 - \beta_0\beta_2\mathbb{S}_0^2\mathbb{V}_0 + \beta_0\alpha_0\mathbb{S}_0\mathbb{E}_0 + \beta_0\mu_0\mathbb{S}_0\mathbb{E}_0 - \beta_0 \wedge \mathbb{E}_0 + (\beta_0)^2\mathbb{S}_0\mathbb{E}_0^2 \\
&+ \beta_0\beta_1\mathbb{S}_0\mathbb{E}_0\mathbb{I}_0 + \beta_0\beta_2\mathbb{S}_0\mathbb{E}_0\mathbb{V}_0 + \beta_0\alpha_0\mathbb{S}_0\mathbb{E}_0 - \beta_1\alpha_0\mathbb{S}_0\mathbb{E}_0 + \beta_1\omega_0\mathbb{S}_0\mathbb{I}_0 + \beta_1\gamma_0\mathbb{S}_0\mathbb{I}_0 + \beta_1\mu_0\mathbb{S}_0\mathbb{I}_0 \\
&- \beta_1 \wedge \mathbb{I}_0 + \beta_0\beta_1\mathbb{S}_0\mathbb{E}_0\mathbb{I}_0 + (\beta_1)^2\mathbb{S}_0\mathbb{I}_0^2 + \beta_1\beta_2\mathbb{S}_0\mathbb{I}_0\mathbb{V}_0 + \beta_1\mu_0\mathbb{S}_0\mathbb{I}_0 - \beta_2\varphi_1\mathbb{S}_0\mathbb{E}_0 - \beta_2\varphi_2\mathbb{S}_0\mathbb{I}_0 \\
&+ \beta_2\sigma_0\mathbb{S}_0\mathbb{V}_0 - \beta_2 \wedge \mathbb{V}_0 - \beta_0\beta_2\mathbb{S}_0\mathbb{E}_0\mathbb{V}_0 + \beta_1\beta_2\mathbb{S}_0\mathbb{I}_0\mathbb{V}_0 + (\beta_2)^2\mathbb{S}_0\mathbb{V}_0^2 + 2\beta_2\mu_0\mathbb{S}_0\mathbb{V}_0 \\
&- \mu_0 \wedge + \beta_0\mu_0\mathbb{S}_0\mathbb{E}_0 + \beta_1\mu_0\mathbb{S}_0\mathbb{I}_0 + \mu_0^2\mathbb{S}_0]\frac{t^2}{2} - [(\beta_0)^2 \wedge \mathbb{S}_0\mathbb{E}_0 + \beta_0\beta_1 \wedge \mathbb{S}_0\mathbb{I}_0 \\
&+ \beta_0\beta_2 \wedge \mathbb{S}_0\mathbb{V}_0 - \beta_0\alpha_0 \wedge \mathbb{E}_0 - \beta_0\mu_0 \wedge \mathbb{E}_0 - (\beta_0)^3\mathbb{S}_0^2\mathbb{E}_0^2 - (\beta_0)^2\beta_1\mathbb{S}_0^2\mathbb{E}_0\mathbb{I}_0 - (\beta_0)^2\beta_2\mathbb{S}_0^2\mathbb{E}_0\mathbb{V}_0 \\
&+ (\beta_0)^2\alpha_0\mathbb{S}_0\mathbb{E}_0^2 + (\beta_0)^2\mu_0\mathbb{S}_0\mathbb{E}_0^2 - (\beta_0)^2\beta_1\mathbb{S}_0^2\mathbb{E}_0\mathbb{I}_0 - \beta_0(\beta_1)^2\mathbb{S}_0^2\mathbb{I}_0^2 \\
&- \beta_0\beta_1\beta_2\mathbb{S}_0^2\mathbb{I}_0\mathbb{V}_0 + \beta_0\beta_1\alpha_0\mathbb{S}_0\mathbb{E}_0\mathbb{I}_0 + \beta_0\beta_1\mu_0\mathbb{S}_0\mathbb{E}_0\mathbb{I}_0 - (\beta_0)^2\beta_2\mathbb{S}_0^2\mathbb{E}_0\mathbb{V}_0 \\
&- \beta_0\beta_1\beta_2\mathbb{S}_0^2\mathbb{I}_0\mathbb{V}_0 - \beta_0(\beta_2)^2\mathbb{S}_0^2\mathbb{V}_0^2 + \beta_0\beta_2\alpha_0\mathbb{S}_0\mathbb{E}_0\mathbb{V}_0 + \beta_0\beta_2\mu_0\mathbb{S}_0\mathbb{E}_0\mathbb{V}_0 \\
&- (\beta_0)^2\mu_0\mathbb{S}_0^2\mathbb{E}_0 - \beta_0\beta_1\mu_0\mathbb{S}_0^2\mathbb{I}_0 - \beta_0\beta_2\mu_0\mathbb{S}_0^2\mathbb{V}_0 + \alpha_0\mu_0\mathbb{S}_0\mathbb{E}_0 + \mu_0^2\mathbb{S}_0\mathbb{E}_0 \\
&+ \beta_1\alpha_0 \wedge \mathbb{E}_0 - \beta_1\omega_0 \wedge \mathbb{I}_0 - \beta_1\gamma_0 \wedge \mathbb{I}_0 - \beta_1\mu_0 \wedge \mathbb{I}_0 - \beta_1\beta_0\alpha_0\mathbb{S}_0\mathbb{E}_0^2 + \\
&\beta_1\beta_0\omega_0\mathbb{S}_0\mathbb{E}_0\mathbb{I}_0 + \beta_1\beta_0\gamma_0\mathbb{S}_0\mathbb{E}_0\mathbb{I}_0 + \beta_1\beta_(\mathbb{E})\mu_0\mathbb{S}_0\mathbb{E}_0\mathbb{I}_0 - (\beta_1)^2\alpha_0\mathbb{S}_0\mathbb{E}_0\mathbb{I}_0 \\
&+ (\beta_1)^2\omega_0\mathbb{S}_0\mathbb{I}_0^2 + (\beta_1)^2\gamma_0\mathbb{S}_0\mathbb{I}_0^2 + (\beta_1)^2\omega_0\mathbb{S}_0\mathbb{I}_0^2 - \beta_1\beta_2\alpha_0\mathbb{S}_0\mathbb{E}_0\mathbb{V}_0 + \beta_1\beta_2\omega_0 \\
&\mathbb{S}_0\mathbb{I}_0\mathbb{V}_0 + \beta_1\beta_2\gamma_0\mathbb{S}_0\mathbb{I}_0\mathbb{V}_0 + \beta_1\beta_2\mu_0\mathbb{S}_0\mathbb{I}_0\mathbb{V}_0 \\
&- \beta_1\alpha_0\mu_0\mathbb{S}_0\mathbb{E}_0 + \beta_1\mu_0\omega_0\mathbb{S}_0\mathbb{I}_0 + \beta_1\mu_0\gamma_0\mathbb{S}_0\mathbb{I}_0 + \beta_1\mu_0^2\mathbb{S}_0\mathbb{I}_0 \\
&+ \beta_2 \wedge \varphi_1\mathbb{E}_0 + \beta_2 \wedge \varphi_2\mathbb{I}_0 - \beta_2 \wedge \sigma_0\mathbb{V}_0 - \beta_2\beta_0\varphi_1\mathbb{S}_0\mathbb{E}_0^2 - \beta_2\beta_0\varphi_2\mathbb{S}_0\mathbb{E}_0\mathbb{I}_0 \\
&+ \beta_2\sigma_0\mathbb{S}_0\mathbb{E}_0\mathbb{V}_0 - \beta_1\beta_2\varphi_1\mathbb{S}_0\mathbb{E}_0\mathbb{I}_0 - \beta_1\beta_2\varphi_2\mathbb{S}_0\mathbb{I}_0^2 \\
&+ \beta_1\beta_2\sigma_0\mathbb{S}_0\mathbb{I}_0\mathbb{V}_0 - (\beta_2)^2\varphi_1\mathbb{S}_0\mathbb{E}_0\mathbb{V}_0 - (\beta_2)^2\varphi_2\mathbb{S}_0\mathbb{I}_0\mathbb{V}_0 \\
&+ (\beta_2)^2\sigma_0\mathbb{S}_0\mathbb{V}_0^2 - \beta_2\mu_0\varphi_1\mathbb{S}_0\mathbb{E}_0 - \beta_2\mu_0\varphi_2\mathbb{S}_0\mathbb{I}_0 + \beta_2\mu_0\sigma_0\mathbb{S}_0\mathbb{V}_0]\frac{t^3}{3}.
\end{aligned} \tag{7}$$

$$\begin{aligned}
&\mathbb{E}_2(t) = \mathbb{E}_0 + [\beta_0\mathbb{S}_0\mathbb{E}_0 + \beta_1\mathbb{S}_0\mathbb{I}_0 + \beta_2\mathbb{S}_0\mathbb{V}_0 - \alpha_0\mathbb{I}_0 - \mu_0\mathbb{I}_0]t - [-(\beta_0)^2\mathbb{S}_0^2\mathbb{E}_0 - \beta_0\beta_1\mathbb{S}_0^2\mathbb{I}_0 - \beta_0\beta_2\mathbb{S}_0^2\mathbb{V}_0 \\
&+ \beta_0\alpha_0\mathbb{S}_0\mathbb{E}_0 + \beta_0\mu_0\mathbb{S}_0\mathbb{E}_0 - \beta_0 \wedge \mathbb{E}_0 + (\beta_0)^2\mathbb{S}_0\mathbb{E}_0^2 + \beta_0\beta_1\mathbb{S}_0\mathbb{E}_0\mathbb{I}_0 \\
&+ \beta_0\beta_2\mathbb{S}_0\mathbb{E}_0\mathbb{V}_0 + \beta_0\alpha_0\mathbb{S}_0\mathbb{E}_0 - \beta_1\alpha_0\mathbb{S}_0\mathbb{E}_0 + \beta_1\omega_0\mathbb{S}_0\mathbb{I}_0 + \beta_1\gamma_0\mathbb{S}_0\mathbb{I}_0 \\
&+ \beta_1\mu_0\mathbb{S}_0\mathbb{I}_0 - \beta_1 \wedge \mathbb{I}_0 + \beta_0\beta_1\mathbb{S}_0\mathbb{E}_0\mathbb{I}_0 + (\beta_1)^2\mathbb{S}_0\mathbb{I}_0^2 + \beta_1\beta_2\mathbb{S}_0\mathbb{I}_0\mathbb{V}_0 + \beta_1\mu_0 \\
&\mathbb{S}_0\mathbb{I}_0 - \beta_2\varphi_1\mathbb{S}_0\mathbb{E}_0 - \beta_2\varphi_2\mathbb{S}_0\mathbb{I}_0 + \beta_2\sigma_0\mathbb{S}_0\mathbb{V}_0 - \beta_2 \wedge \mathbb{V}_0 - \beta_0\beta_2\mathbb{S}_0\mathbb{E}_0\mathbb{V}_0 + \beta_1\beta_2\mathbb{S}_0\mathbb{I}_0\mathbb{V}_0 \\
&+ (\beta_2)^2\mathbb{S}_0\mathbb{V}_0^2 + \beta_2\mu_0\mathbb{S}_0\mathbb{V}_0 + \alpha_0^2\mathbb{E}_0 - \alpha_0\omega_0\mathbb{I}_0 - \alpha_0\gamma_0\mathbb{I}_0 - \alpha_0\mu_0\mathbb{I}_0 + \alpha_0\mu_0\mathbb{E}_0 \\
&- \mu_0\omega_0\mathbb{I}_0 - \mu_0\gamma_0\mathbb{I}_0 - \mu_0^2\mathbb{I}_0]\frac{t^2}{2} - [(\beta_0)^2 \wedge \mathbb{S}_0\mathbb{E}_0 + \beta_0\beta_1 \wedge \mathbb{S}_0\mathbb{I}_0 + \beta_0\beta_2 \wedge \mathbb{S}_0\mathbb{V}_0 - \beta_0\alpha_0 \wedge \mathbb{E}_0 - \beta_0\mu_0 \wedge \mathbb{E}_0 \\
&- (\beta_0)^3\mathbb{S}_0^2\mathbb{E}_0^2 - (\beta_0)^2\beta_1\mathbb{S}_0^2\mathbb{E}_0\mathbb{I}_0 - (\beta_0)^2\beta_2\mathbb{S}_0^2\mathbb{E}_0\mathbb{V}_0 + (\beta_0)^2\alpha_0\mathbb{S}_0\mathbb{E}_0^2 \\
&+ (\beta_0)^2\mu_0\mathbb{S}_0\mathbb{E}_0^2 - (\beta_0)^2\beta_1\mathbb{S}_0^2\mathbb{E}_0\mathbb{I}_0 - \beta_0(\beta_1)^2\mathbb{S}_0^2\mathbb{I}_0^2 \\
&- \beta_0\beta_1\beta_2\mathbb{S}_0^2\mathbb{I}_0\mathbb{V}_0 + \beta_0\beta_1\alpha_0\mathbb{S}_0\mathbb{E}_0\mathbb{I}_0 + \beta_0\beta_1\mu_0\mathbb{S}_0\mathbb{E}_0\mathbb{I}_0 \\
&- (\beta_0)^2\beta_2\mathbb{S}_0^2\mathbb{E}_0\mathbb{V}_0 - \beta_0\beta_1\beta_2\mathbb{S}_0^2\mathbb{I}_0\mathbb{V}_0 - \beta_0(\beta_2)^2\mathbb{S}_0^2\mathbb{V}_0^2 \\
&+ \beta_0\beta_2\alpha_0\mathbb{S}_0\mathbb{E}_0\mathbb{V}_0 + \beta_0\beta_2\mu_0\mathbb{S}_0\mathbb{E}_0\mathbb{V}_0 - (\beta_0)^2\mu_0\mathbb{S}_0^2\mathbb{E}_0
\end{aligned} \tag{8}$$

$$
\begin{aligned}
&-\beta_0\beta_1\mu_0\mathbb{S}_0^2\mathbb{I}_0-\beta_0\beta_2\mu_0\mathbb{S}_0^2\mathbb{V}_0+\alpha_0\mu_0\mathbb{S}_0\mathbb{E}_0+\mu_0^2\mathbb{S}_0\mathbb{E}_0+\beta_1\alpha_0\wedge\mathbb{E}_0\\
&-\beta_1\omega_0\wedge\mathbb{I}_0-\beta_1\gamma_0\wedge\mathbb{I}_0-\beta_1\mu_0\wedge\mathbb{I}_0-\beta_1\beta_0\alpha_0\mathbb{S}_0\mathbb{E}_0^2+\beta_1\beta_0\omega_0\mathbb{S}_0\mathbb{E}_0\mathbb{I}_0\\
&+\beta_1\beta_0\gamma_0\mathbb{S}_0\mathbb{E}_0\mathbb{I}_0+\beta_1\beta_(\mathbb{E})\mu_0\mathbb{S}_0\mathbb{E}_0\mathbb{I}_0-(\beta_1)^2\alpha_0\mathbb{S}_0\mathbb{E}_0\mathbb{I}_0+(\beta_1)^2\omega_0\mathbb{S}_0\mathbb{I}_0^2\\
&+(\beta_1)^2\gamma_0\mathbb{S}_0\mathbb{I}_0^2+(\beta_1)^2\omega_0\mathbb{S}_0\mathbb{I}_0^2-\beta_1\beta_2\alpha_0\mathbb{S}_0\mathbb{E}_0\mathbb{V}_0+\beta_1\beta_2\omega_0\mathbb{S}_0\mathbb{I}_0\mathbb{V}_0\\
&+\beta_1\beta_2\gamma_0\mathbb{S}_0\mathbb{I}_0\mathbb{V}_0+\beta_1\beta_2\mu_0\mathbb{S}_0\mathbb{I}_0\mathbb{V}_0-\beta_1\alpha_0\mu_0\mathbb{S}_0\mathbb{E}_0+\beta_1\mu_0\omega_0\mathbb{S}_0\mathbb{I}_0\\
&+\beta_1\mu_0\gamma_0\mathbb{S}_0\mathbb{I}_0+\beta_1\mu_0^2\mathbb{S}_0\mathbb{I}_0+\beta_2\wedge\varphi_1\mathbb{E}_0+\beta_2\wedge\varphi_2\mathbb{I}_0-\beta_2\wedge\sigma_0\mathbb{V}_0\\
&-\beta_2\beta_0\varphi_1\mathbb{S}_0\mathbb{E}_0^2-\beta_2\beta_0\varphi_2\mathbb{S}_0\mathbb{E}_0\mathbb{I}_0+\beta_2\sigma_0\mathbb{S}_0\mathbb{E}_0\mathbb{V}_0\\
&-\beta_1\beta_2\varphi_1\mathbb{S}_0\mathbb{E}_0\mathbb{I}_0-\beta_1\beta_2\varphi_2\mathbb{S}_0\mathbb{I}_0^2+\beta_1\beta_2\sigma_0\mathbb{S}_0\mathbb{I}_0\mathbb{V}_0\\
&-(\beta_2)^2\varphi_1\mathbb{S}_0\mathbb{E}_0\mathbb{V}_0-(\beta_2)^2\varphi_2\mathbb{S}_0\mathbb{I}_0\mathbb{V}_0+(\beta_2)^2\sigma_0\mathbb{S}_0\mathbb{V}_0^2\\
&-\beta_2\mu_0\varphi_1\mathbb{S}_0\mathbb{E}_0-\beta_2\mu_0\varphi_2\mathbb{S}_0\mathbb{I}_0+\beta_2\mu_0\sigma_0\mathbb{S}_0\mathbb{V}_0]\frac{t^3}{3}.
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{I}_2(t)=&\ \mathbb{I}_0+[\alpha_0\mathbb{E}_0-\omega_0\mathbb{I}_0-\gamma_0\mathbb{I}_0-\mu_0\mathbb{I}_0]t+[-\alpha_0\omega_0\mathbb{E}_0+\omega_0^2\mathbb{I}_0+2\omega_0\gamma_0\mathbb{I}_0+2\omega_0\mu_0\mathbb{I}_0-\alpha_0\mu_0\mathbb{E}_0\\
&+\gamma_0^2\mathbb{I}_0-\alpha_0\mu_0\mathbb{E}_0+\mu_0\gamma_0\mathbb{I}_0+\mu_0^2\mathbb{I}_0+\beta_0\alpha_0\mathbb{S}_0\mathbb{E}_0+\alpha_0(\mathbb{I})\alpha_0\mathbb{S}_0\mathbb{I}_0+\beta_2\alpha_0\mathbb{S}_0\mathbb{V}_0-\alpha_0^2\mathbb{E}_0]\frac{t^2}{2}.
\end{aligned}
\tag{9}
$$

$$
\mathbb{R}_2(t)=\mathbb{R}_0+[\gamma_0\mathbb{I}_0-\mu_0\mathbb{R}_0]t+[-2\gamma_0\mu_0\mathbb{I}_0+\mu_0^2\mathbb{R}_0+\alpha_0\gamma_0\mathbb{E}_0-\gamma_0\omega_0\mathbb{I}_0-\gamma_0^2\mathbb{I}_0]\frac{t^2}{2}. \tag{10}
$$

We can compute the other terms in similar way.

5. Numerical Discussion and Graphical Representations

To present the concerned approximate solutions computed above of the model under consideration, we recall some numerical values for the parameters in the given table 1. Based on reported data the initial condition is set as [39]

$$
\Big(\mathbb{S}(0),\mathbb{E}(0),\mathbb{I}(0),\mathbb{R}(0)\Big)=(32.37\times10^6,12\times10^6,0.001523\times10^6,0.005025\times10^6).
$$

Parameters	Description of Parameters	Numerical value
γ	tested negative population	0.250281×10^6
η	tested positive population	0.006656×10^6
k	the infection rate	0.000024
α	rate of individual lose immunity	0.01182
ε	natural death rate	0.0000004×10^6
υ	death rate due to C0VID-19	0.016
δ	infected rate	0.025
β	recovered rate	0.75

Table 1. Description of the parameters of the Model (1).

After putting the numerical values in equation (6) we obtained the following results.

CASE (1) N=0

$$\begin{cases} \mathbb{S}_1(t) = 8998505 - (1232.8251)t, \\ \mathbb{E}_1(t) = 1000 + (1090.31287)t, \\ \mathbb{I}_1(t) = 475 + (106.4262)t, \\ \mathbb{R}_1(t) = 10 + (31.6663)t. \end{cases} \tag{11}$$

And similarly from equation (7),(8),(9), (10) and (11), we get

CASE (2) N=1

$$\begin{cases} \mathbb{S}_2(t) = 8998505 - (1232.8251)t + (201.3425)t^2 - (0.0184)t^3, \\ \mathbb{E}_2(t) = 1000 + (1090.3129)t - (279.2834)t^2 - (0.0184)t^3, \\ \mathbb{I}_2(t) = 475 + (106.4262)t + (73.7982)t^2, \\ \mathbb{R}_2(t) = 10 + (31.6664)t + (3.5471)t^2. \end{cases} \tag{12}$$

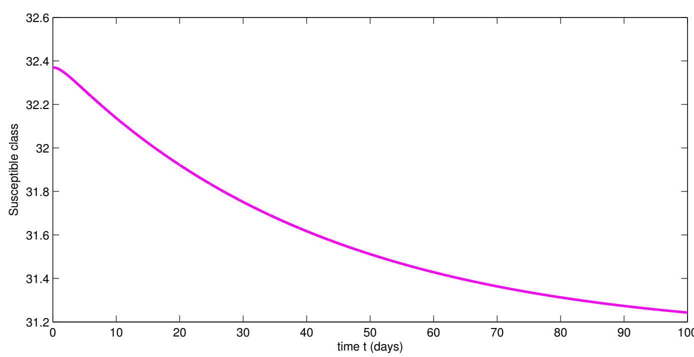

FIGURE 1. The Dynamical behavior of Susceptible Class

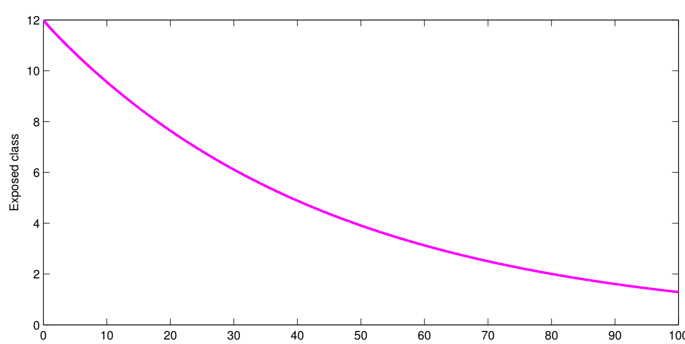

FIGURE 2. The Dynamical behavior of Exposed Class

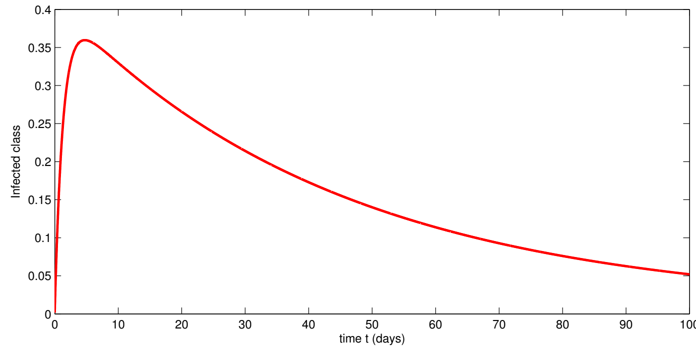

FIGURE 3. The Dynamical behavior of Infected Class

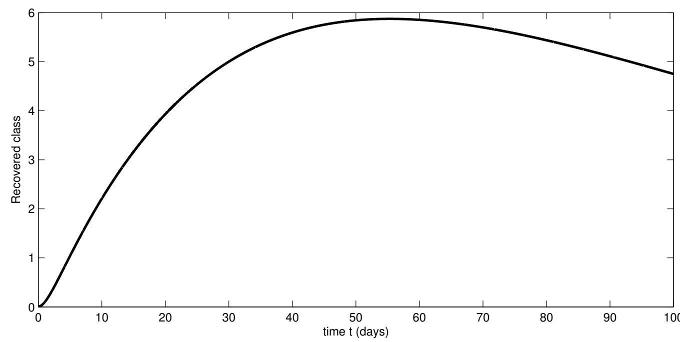

FIGURE 4. The Dynamical behavior of Recovered Class

From fig 1, we see that as the susceptible class decreases, which result in increases in infected class as in fig 3. But due to vaccination and SOP's there is increased in recovered class as in fig 4.

6. **Concluding remarks**

We have investigated a mathematical model of COVID-19 under VIM. By using the said technique, some numerical results in the form of infinite series have been studied. The concerned results along with their graphical representation via MATLAB were given. We can see that VIM key benefits lies in its flexibility and straightforwardness in solving nonlinear equations. The approach is applicable to both bounded and unbounded domains. If there is an exact solution to the differential equations, this method can be used to discover the convergent successive approximations of the exact solution.

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